REMARKS

This amendment is responsive to the Final Rejection of October 10, 2006. Reconsideration and allowance of all claims are requested.

The Office Action

The Examiner rejects claims 2, 3, and 5-9 under 35 U.S.C. § 112, first paragraph.

Claims 2 and 6 stand rejected under 35 U.S.C. § 103 as being unpatentable over Arellano (US 2004/0128624) in view of Maissel (US 6,637,029).

Claims 3, 5, and 7-9 stand rejected under 35 U.S.C. § 102 as being anticipated by Arellano.

Claims 1-13, 34-38, and 41 stand rejected under 35 U.S.C. § 103 as being unpatentable over Arellano in view of Akella (US 2002/0178146).

Claims 15, 39, and 42 stand rejected under 35 U.S.C. § 103 as being unpatentable over Arellano in view of Akella, further in view of Elenbaas (US 2005/0028194).

Claims 16, 40, and 43 stand rejected under 35 U.S.C. § 103 as being unpatentable over Arellano, in view of Akella, further in view of Elenbaas, further yet in view of Boloker (US 2002/0194388; US 7,028,306).

Claims 17, 20-22, 24, 25, and 29-33 stand rejected under 35 U.S.C. § 103 as being unpatentable over Arellano in view of Boloker, further in view of Elenbaas.

Claims 18 and 19 stand rejected under 35 U.S.C. § 103 as being unpatentable over Arellano in view of Boloker, further in view of Elenbaas, further yet in view of Sezan (US 2005/0091686).

The Examiner further made objections to claims 2, 21, 22, 31, and 32.

The Present Amendment Should Be Entered

The Boloker reference, which was cited relative to the claimed modal-logic, is actually concerned with multi-modality browsers. Although these two phrases may have linguistic similarity, the concepts which they represent are completely different for the reasons set forth below. If the Examiner chooses to issue a new ground of rejection replacing Boloker, then the *Finality* must be withdrawn, the

present amendment entered, and a *non-Final* Office Action issued. On the other hand, if the Examiner agrees to allow the claims that were previously rejected based on Boloker, then the *Finality* of the prior Office Action need not be withdrawn. However, it is submitted that the amendments to claims 2 and 5 should still be entered. First, these amendments resolve the Examiner's objections and the 35 U.S.C. § 112 issue, reducing the issues on Appeal. Second, the amendment to claim 5 incorporates the allowable modal-logic subject matter and the amendment should be entered as placing claim 5 in condition for allowance. Moreover, because the issue of modal-logic has been considered in conjunction with claims 17-22, 24, 25, and 29-33, it is submitted that adding this concept to claim 5 will require no additional search or consideration.

The 37 CFR 1.131 Declaration could not have been presented sooner because Akella was cited and applied for the first time in the October 10, 2006 Office Action.

The evidence that the Examiner is misconstruing "multi-modal browser" could not be presented earlier because the Examiner cited and applied Boloker for the first time in the October 10, 2006 Office Action.

The Claim Objections

Claim 2 has been amended as suggested by the Examiner. Claims 21, 22, 31, and 32 have been cancelled as suggested by the Examiner.

35 U.S.C. § 112

Claim 5 has been amended to delete the user behavior limitation which the Examiner asserted is not supported by the original disclosure. Accordingly, it is submitted that claim 5 and claims 2, 3, and 6-9 dependent therefrom now comply with the requirements of 35 U.S.C. § 112.

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Discussion

Multi-Modal Browser v. Modal Logic

The Examiner concedes that Arellano, Akella, and Elenbaas fail to disclose using "modal logic". The Examiner asserts that modal logic is suggested in paragraphs [0081] and [0082] of Boloker. More accurately, Boloker describes a "multi-modal browser" and the description starts at paragraph [0075] and continues through paragraph [0082].

Multi-modal browsers do not use **modal logic**. Rather, **multi-modal browsers** and **modal logic** are unrelated concepts. Boloker, which is assigned to IBM, provides a fairly good discussion regarding multi-modal browsers as relating to a browser for internet access that provides for user interaction with multiple channels and devices which can be used simultaneously to gain sequential or parallel information access. An internet search also developed the enclosed documents "Multimodal Why IBM? – Leadership in Multimodal" and "Multi-Modal Browser Architecture Overview on the Support of Multi-Modal Browsers in 3GPP". The latter of these two documents, in the "Definitions/Summary section", defines "multi-modal applications" as "multi-channel applications, where multiple channels are simultaneously available and synchronized".

Whereas a multi-modal browser is concerned with simultaneously accessing the internet with multiple modalities, modal logic relates to a logic for handling concepts like a possibility, impossibility, necessity, eventually, formerly, can, could, might, must, etc. The Examiner's attention is directed to the enclosed definition of modal-logic from Wikipedia. The applicants further enclose a definition of modal-logic from the Stanford Encyclopedia of Philosophy. Modal-logic is further described in the last paragraph of page 12 of the present application.

These four submissions are made pursuant to MPEP 2111.01 as evidence of the ordinary meaning of modal logic and multi-modal browser. Because these definitional materials are not submitted under 37 CFR 1.97-1.99, no Information Disclosure Statement is enclosed. Moreover, as evidence under MPEP 2111.01, these documents need not be "prior" in the sense of prior art.

The Examiner is invited to use additional dictionaries to gain a better understanding of the differences between modal-logic and multi-modal browser architecture.

Once one understands the meaning of "modal logic" and the meaning of "multi-modal browser" as commonly used, it is clear that the Boloker patent in no way suggests the use of modal logic. Accordingly, it is submitted that **claims 16-22, 24, 25, 29-33, 40, and 43** which call for modal logic are neither taught nor fairly suggested by the multi-modal browser of Boloker. An early allowance of **claims 16-22, 24, 25, 29-33, 40, and 43** are requested.

In addition to amending **claim 5** as suggested by the Examiner to resolve the 35 U.S.C. § 112 issue, the applicants have also amended claim 5 to incorporate subject matter from claim 16, particularly the use of modal logic. Accordingly, it is submitted that **claim 5 and claims 2, 3, and 6-9 dependent therefrom** distinguish patentably and unobviously over the references of record.

The Applicants Invented As Much As Akella Shows Before its Filing Date

In the rejection of **claims 10-13, 15, 16, and 34-44**, the Examiner relies on Akella for the concept of analyzing a series of snapshots in order to detect changes in user information and determine historical trends. The applicants enclose an appropriate 37 CFR 1.131 Affidavit swearing behind Akella.

The Examiner alleges that:

> In an analogous art, Akella teaches it is desirable to analyze a series of snapshots in order to detect changes in user information and determine historical trends (Abstract; Par. 9; Par. 10; lines 1-5; Par. 23, lines 1-6; Par. 24; Par. 27, lines 15-21).

The invention disclosure document attached to the 37 CFR 1.131 Affidavit shows as much as the Akella reference shows (MPEP 15.02; *In re Stryker*, 435 F.2d. 1340, 168 U.S.P.Q. 372 (CCPA 1971). Akella, in paragraphs 9, 10, 23, 24, and 27 referenced by the Examiner, takes snapshots to maintain a historical record which can be referenced at a later time. The paragraphs referenced by the Examiner are primarily concerned with rules for determining when to take a snapshot rather than any analysis of the snapshots. The example given in

paragraph 23 relates to current address and contact information for a customer. Paragraph 25 and the abstract suggest that ad hoc queries **170** can be performed using the historical snapshot data to discover opportunities, trends, or mistakes that may be included within historical snapshots **140**. In paragraph 25, it appears that this data mining process is a mental process by the inquirer. Paragraph 25 and the abstract make no suggestion of any program or processor programmed to discover such trends. Indeed, with the paragraph 23 example of current contact information, there is no suggestion that one should analyze historical snapshots to try to predict future customer contact information.

With the removal of Akella as a reference, it is submitted that **claims 10-13, 15, 16, and 34-44** now distinguish patentably and unobviously over the references of record.

Akella Shows Less Than the Examiner Asserts

Claims 10-13, 15, 16, and 34-44 distinguish patentably over Akella. Akella maintains a snapshot memory which is suitable for ad hoc data mining (paragraph 25). By contrast, claim 10 calls for a processor which is programmed with code to perform a series of operations including analyzing the snapshots for adaptive memory tracking and evolution of the user. Akella's ad hoc data mining does not teach or fairly suggest such a suitably programmed processor.

Claim 34 calls for code for analyzing a plurality of snapshots to develop patterns, trends, and tendencies. Akella uses ad hoc data mining and does not suggest such code. Moreover, claim 34 calls for pushing content to the relevant user in accordance with said patterns, trends, and tendencies developed from the snapshots. Akella makes no suggestion of code which reacts to patterns, trends, and tendencies developed from the series of snapshots. Again, the examples in Akella are ad hoc and performed by individuals. For example, paragraph 11 suggests that if many different clients are moving to another city, it might be a good idea to open a branch office in the other city. The example of paragraph 11 of Akella analyzes snapshots of different users and reacts by opening a branch office and not by pushing content by computer.

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Accordingly, it is submitted that **claims 10 and 34, and claims 11-13, 15, 16, and 35-44 dependent therefrom** distinguish patentably and unobviously over the references of record.

CONCLUSION

For the reasons set forth above, it is submitted that all claims now comply with the statutory requirements and distinguish patentably and unobviously over the references of record. An early allowance of all claims is requested.

Respectfully submitted,

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IBM

Multimodal

Why IBM? - Leadership in multimodal.

Overview

As devices become smaller, modes of interaction other than keyboard and stylus are a necessity. In particular, small handheld devices like cell phones and PDAs serve many functions and contain sufficient processing power to handle a variety of tasks. Present and future devices will greatly benefit from the use of multimodal access methods.

Multichannel access is the ability to access enterprise data and applications from multiple methods or channels such as a phone, laptop or PDA. For example, a user may access his or her bank account balances on the Web using Microsoft® Internet Explorer when in the office or at home and may access the same information over a regular phone using voice recognition and text-to-speech when on the road.

By contrast, multimodal access is the ability to combine multiple modes or channels in the same interaction or session. The methods of input include speech recognition, keyboard, touch screen, and stylus. Depending on the situation and the device, a combination of input modes will make using a small device easier. For example, in a Web browser on a PDA, you can select items by tapping or by providing spoken input. Similarly, you can use voice or stylus to enter information into a field. With multimodal technology, information on the device can be both displayed and spoken.

Motorola, Opera Software ASA, and IBM submitted to the W3C® a proposal for a multimodal markup language standard called XHTML+Voice (X+V for short) that provides a way to create multimodal Web applications (i.e., Web applications that offer both a voice and visual interface).

Multimodal applications using X+V offer a natural migration path from today's VoiceXML-based voice applications and XHTML-based visual applications to a single application that can serve both of these environments as well as multimodal ones (refer to Figure 1).

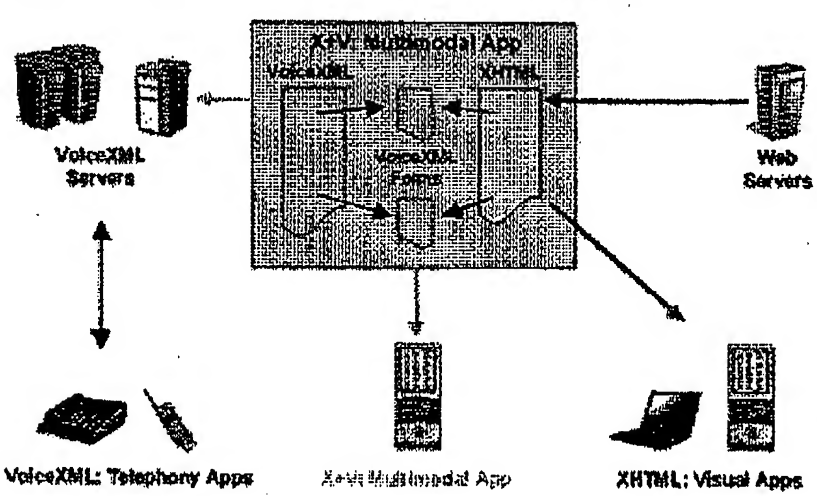

Figure 1: One application accessed by many devices.

Experience IBM multimodal demos based on the X+V specification.

Multimodal Browsers for Embedded Devices

The Multimodal Browsers have been developed in a strategic relationship with Opera Software (based on the Opera Browser V7.55) and ACCESS Systems Company (based on the NetFront Browser V3.1 by ACCESS Systems). They are each enhanced with extensions that include the IBM speech recognition and text-to-speech technology, allowing you to view and interact with multimodal applications built using XHTML+Voice.

When you install the Multimodal Browser, the icons for the browsers appear on your device, and you can use the icons to open the browsers and run your multimodal application.

ACCESS Systems' NetFront Multimodal Browser for PocketPC 2003	Download now
Opera Software Multimodal Browser for Sharp Zaurus	Download now

Multimodal Tools are available now

IBM released the latest version of Multimodal Tools. These tools can simplify multimodal application development and work seamlessly with the IBM WebSphere® family of products.

The Multimodal Tools V4.1.4 Refresh includes:

- Multimodal Toolkit V6.0 for Rational Web Developer 6.0.1.1
- Multimodal Browser V4.1.4

Download now

Additional pervasive application development tools can be found in the Pervasive section at the DevX web site. This section contains additional downloads, tutorials and upcoming events.

For more information

For full details on X+V, please refer to the W3C Multimodal Interaction and the VoiceXML Forum web sites.

Related Articles:

- Talk to your TV: Opera Software Announces Voice-Enabled Home Media Technology with IBM Embedded ViaVoice
- IBM and Teges Corporation Partner to Provide Pediatric ICU with Software
- IBM WebSphere Making The Rounds At Miami Children's Hospital
- Talking to your TV
- Adding Voice to XHTML, Dr. Dobb's Journal, January 2005 (Reg. req'd)
- Look Ma Bell - No Hands! : VoiceXML, X+V, and the mobile device
- ACCESS Announces NetFront v3.2, the World's Most Advanced Browser for Mobile and Home Internet Devices

- Opera Finds Its Voice
- Intermec Offers Robust Speech Recognition Capabilities for 700 Series Color Mobile Computers, Using IBM ViaVoice®
- ACCESS Launches Integrated Mobile Browser/Voice Solution Based on IBM Software
- New IBM Technologies Ease Wireless Deployment
- CNET: IBM unveils toolkit for talking computers
- InternetNews:IBM Takes Wraps Off Multimodal Toolkit
- Wireless Developer Network: IBM Announces Multimodal Toolkit for WebSphere Studio
- Linuxdevices.com: Multimodal browser technology to bring XHTML+Voice to devices
- InternetNews: Opera 7 Holes Detected; Multimodal Toolkit Released
- Communications Convergence: Multimodal Toolkits: Design, In - Speech Tags and ML, Out
- Speech Technology Magazine: IBM Announces Growing Developer Support for Multimodal Applications
- Serverworld: IBM will deliver multimodal toolkit
- IBM Gives Voice To Mobile Communications Technologies
- IBM: Multimodal Applications: Another step in computer/human interaction

IBM Research

Mobile Speech Solutions and Conversational Multi-modal Computing

Multi-Modal Browser Architecture

Overview on the support of multi-modal browsers in 3GPP

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OUTLINE

- Motivation: Multi-modal Mobile e-Business
  - Introduction
  - Multi-channel and pain points
  - Multi-modal and value proposition
  - Definitions
- Recommended architecture
  - MVC Principle
  - DOM-Based MVC multi-modal and multi-device browsers
  - Supported configurations
- What is needed?
  - Infrastructure
  - Protocols, Interfaces and Components to be Standardize
  - DSR and Multi-modal Protocol stack for 3GPP
- Conclusions

Motivation: Multi-modal Mobile e-Business

Introduction - Multi-modal Browser

- **Modality:** A particular type physical interface that can be perceived or interacted with by the user (e.g. voice interface, GUI display with keypad etc...)
- **Multi-modal Browser:**
  - A browser that enables the user to interact with an application through different modes of intercation (e.g. typically: Voice and GUI).
  - Accordingly a multi-modal-browser provides different moadlities for input and output
  - Ideally it lets the user select at any time the modality that is the most appropriate to perform a particular interaction given this interaction and the users situation (activity, environment etc...)
- **Thesis:** By improving the user interface, we believe that multi-modal browsing will significantly accelerate the acceptance and growth of m-Commerce.

1ulti-channel scenario: travel reservations

- Same application can be adapted to different channels
- Synchronization across different channels is needed but more complex.

- Multiple access mechanisms
- One interaction mode per device

PC

Flights Hotels Cars Packages Cruises Maps

EXPRESS SEARCH

Departing from: [ ] **Going to:** [ ]

When are you leaving? [Dec] [31] [Noon] **When are you returning?** [Jan] [1] [Noon]

Tip: We have many more flight, hotel, and car options.

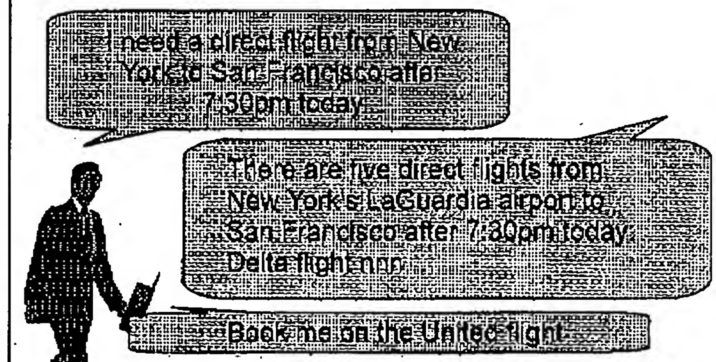
WHAT'S NEW

Ski Travel: Choose from more than 80 ski destinations

Cruise Travel: Take a virtual tour of select cruise ships

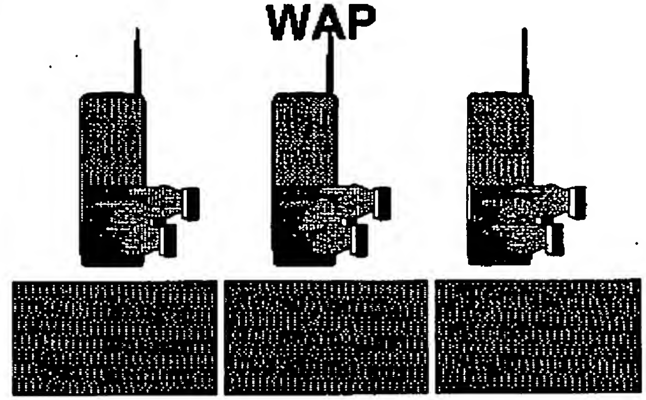
+ Standardized rich visual interface
– Not suitable for mobile use

Voice



+ Access from any telephone
– Output is inherently sequential

WAP



+ Mobile and becoming ubiquitous
– Hard to enter data

Pain points in multi-channel e-business

Most mobile device usage today is not for e-business applications

Pain points

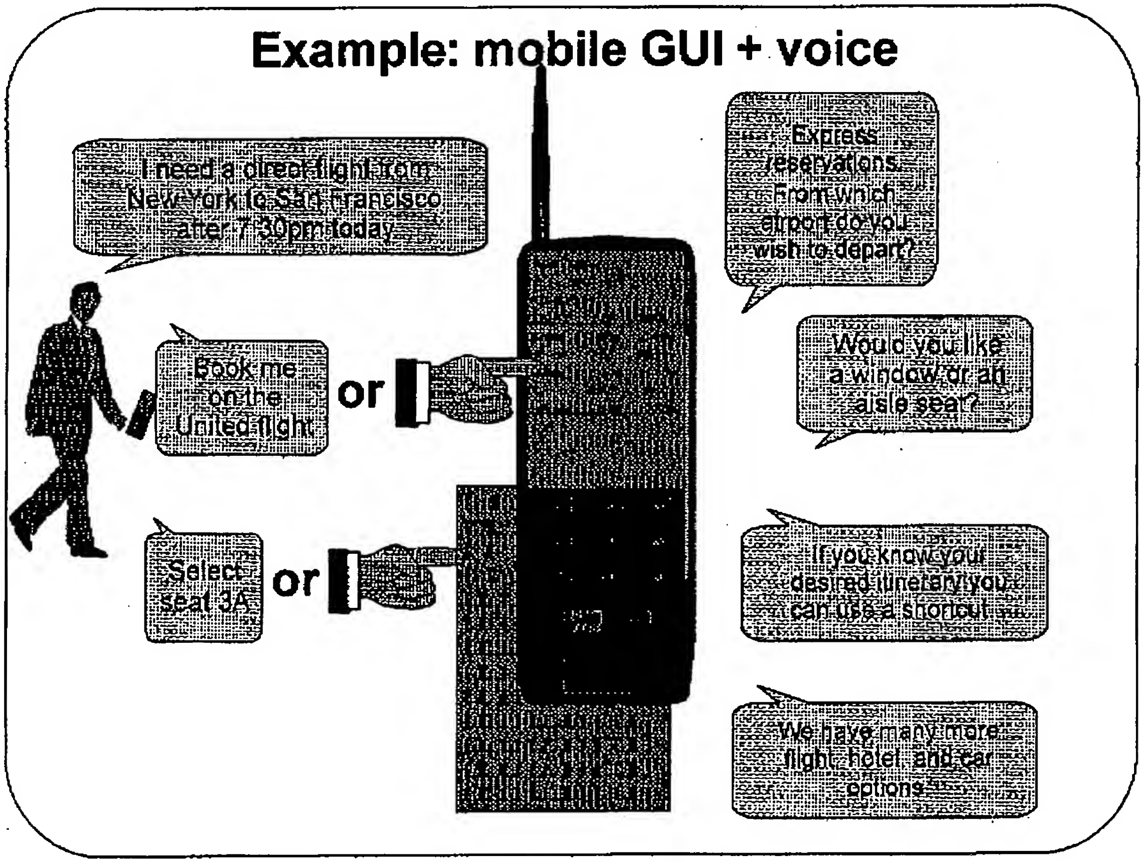
- **hard to enter and access data using small devices**
  - tiny keypads and screens
- **Voice Recognition still makes mistakes**
  - blocking if repeated
- **Voice is serial**
  - difficult to manage long output
- **one interaction mode does not suit all circumstances**
  - each mode has its pros and cons
- **all-in-one devices are no panacea**
  - bulky and expensive
- **multiple devices have pros and cons**

No immediate relief is in sight:

- Devices are getting smaller, not larger
- Devices and applications are becoming more complex
- Adding color, animation, camera, etc. does not simplify or contribute to e-business
- CRMs / IVRs are mostly not yet web-centric

Multi-modal scenario: travel reservation

- User can select at any time the preferred modality of interaction
- Can be extended to selection of the preferred device (multi-device)



Additional examples:

- display seat selection chart (not simply "window or aisle")
- use voice or keys to enter PIN code and performs speaker verification
- use audio or voice for notifications
- information can be saved for later use

- User is not tied to a particular channel's presentation flow
- Interaction becomes a personal and optimized experience
- Multi-modal output is an example of multi-media where the different modalities are closely synchronized.

Multi-modal e-business value proposition

Multi-modal e-business value proposition

- easily enter and access data using small devices
  - by combining multiple input & output modes
- choose at any time the interaction mode that suits the task and circumstances
  - input: key, touch, stylus, voice...
  - output: display, tactile, audio...
  - don't be blocked by limitation / mistales of a given interaction mode at a given moment
- use several devices in combination
  - by exploiting the resources of multiple devices

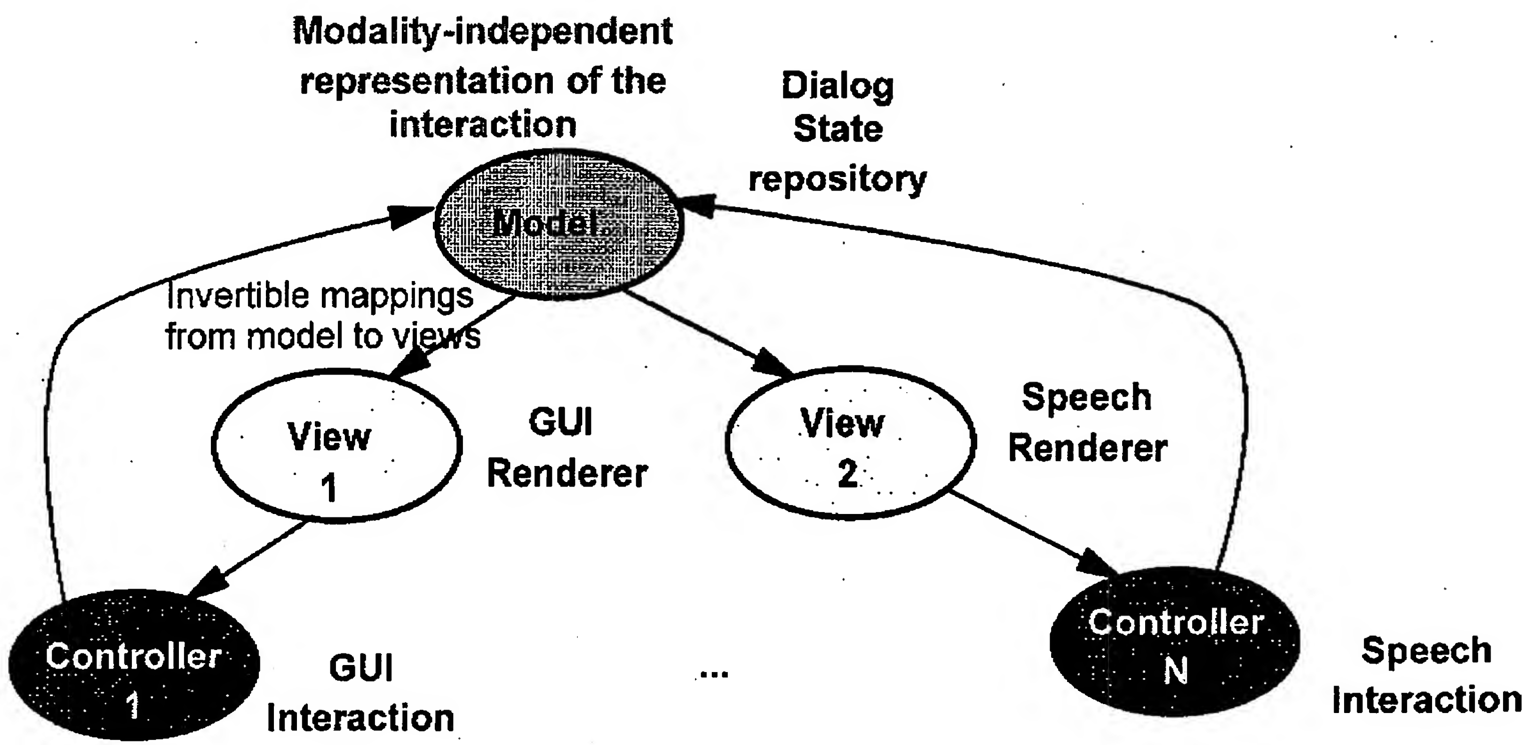
Definitions - Summary

- **Channel:** a particular user agent, device, or a particular modality. Do not confuse this is not a physical communication channel. It is rather typpically the browser / user agent used to access, browse and interact with online information
- **Multi-channel applications**: applications designed for ubiquitous access through different channels, one channel at a time. No particular attention is paid to synchronization or coordination across different channels.
- **Multi-modal applications:** multi-channel applications, where multiple channels are simultaneously available and synchronized.
  - There are no fundamental differences between **multiple devices (multi-device browsing**) and **multiple modalities.**

Recommended Architecture

Model View Controller Principle (MVC)

User must be able to switch channel at any unpredictable moment while interacting with the application and seamlessly continue to interact

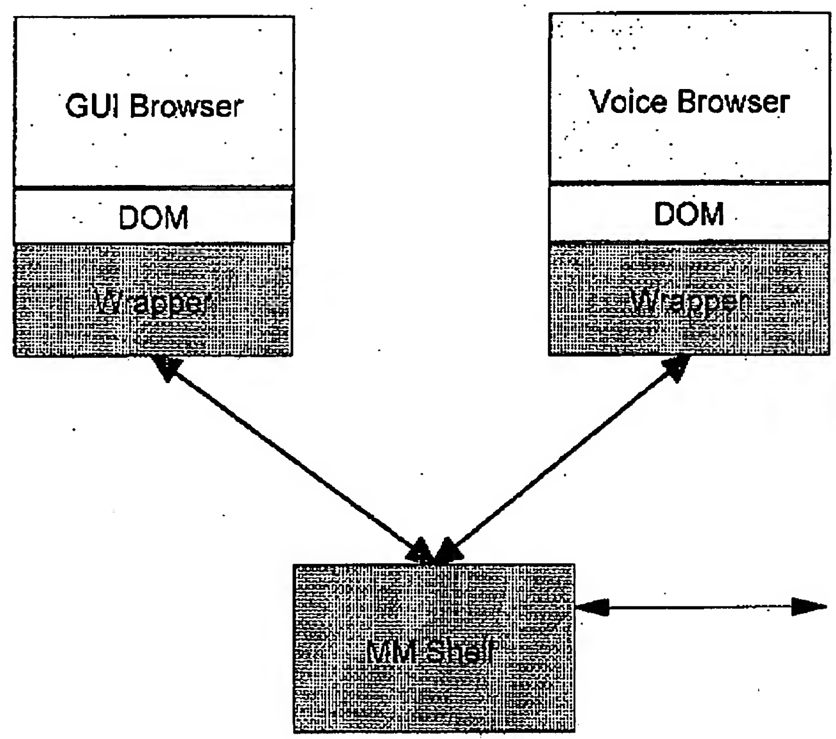


Model View Controller Architecture for Multi-modal or Multi-device Browser

- DOM: Document Object Model (http://www.w3c.org/dom).
- Adapted definitions:
- DOM L1: Interface that enables manipulation of the XML document in each browser
- DOM L2: Interface that provides access to the eventsassociated to the user interaction within each browser

Target: DOM-based architecture

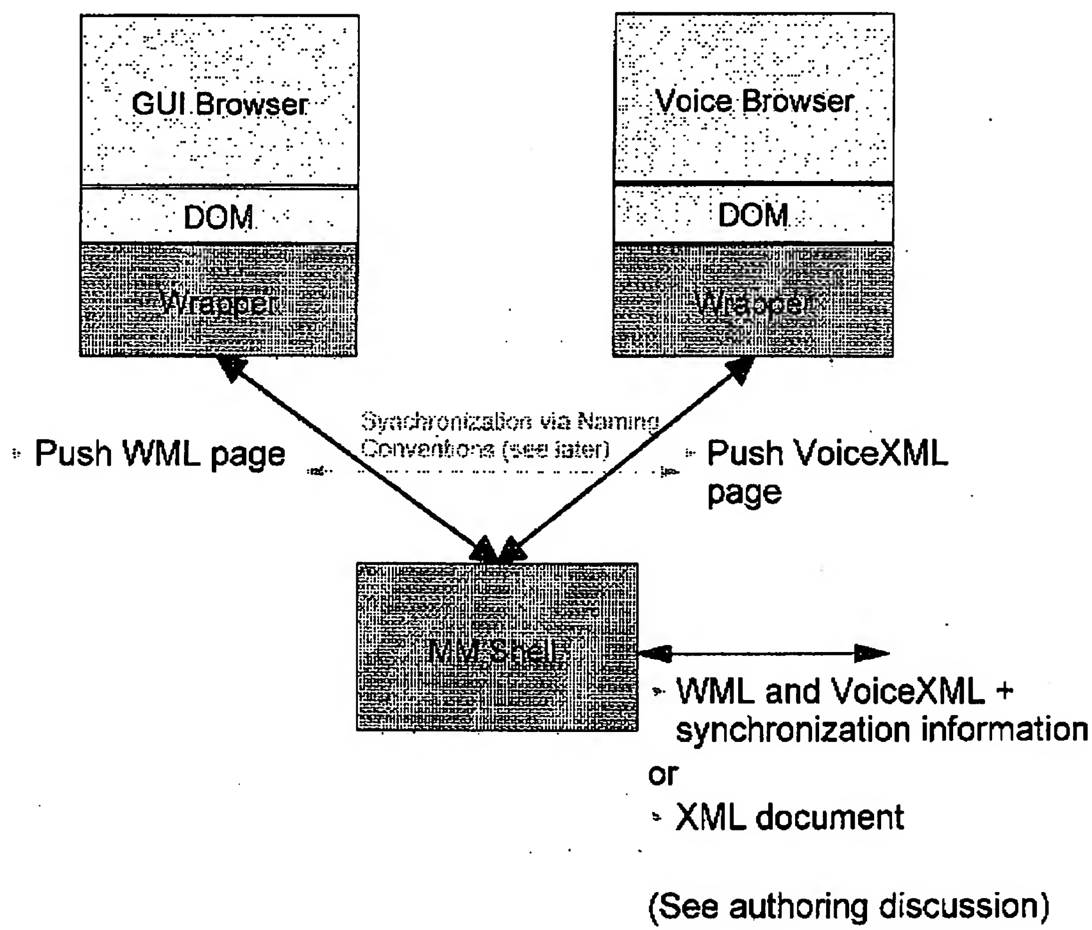
Each piece is distributable



This can be another browser in the case of multi-device browsing

WAP MVC Multi-modal Browser

Initialization



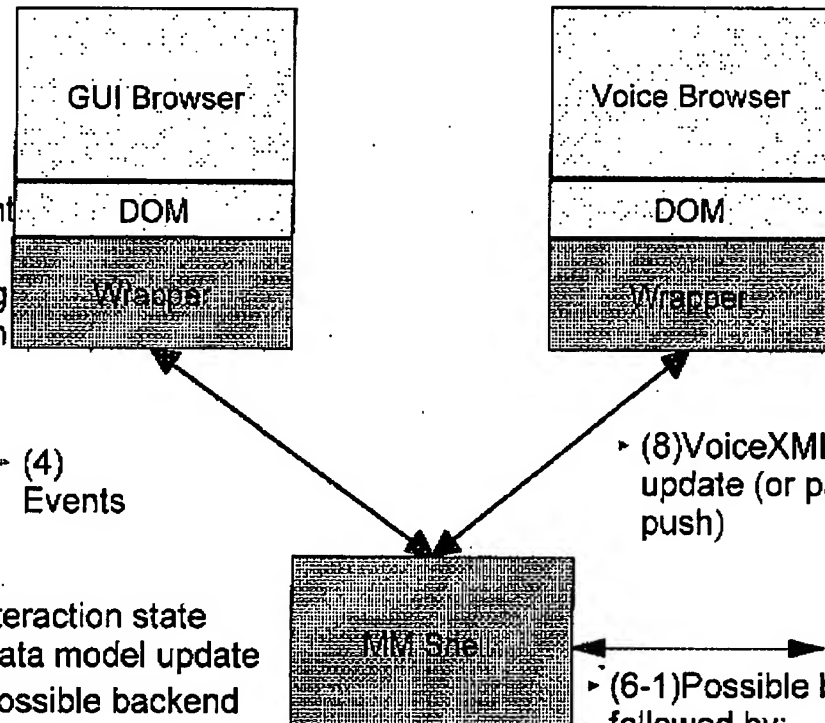
WAP MVC Multi-modal Browser

Interaction: Assuming GUI interaction

(10)Possible additional WML page and browser update
(9)Possible WML DOM command

(8)Possible WML DOM update (or page push)

- (1)User action
- (2)DOM L2 event
- (3) Event filtering and transmission



- (4) Events

- (10)VoiceXML page and browser update
- (9)VoiceXML DOM command
- (8)VoiceXML DOM update (or page push)

- (5) Interaction state and data model update
- (6) (possible backend fetch)
- (7)Update command for registered views

- (6-1)Possible backend submit or request followed by:
  - (6-2)WML and VoiceXML + synchronization information

or

  - (6-2)XML document

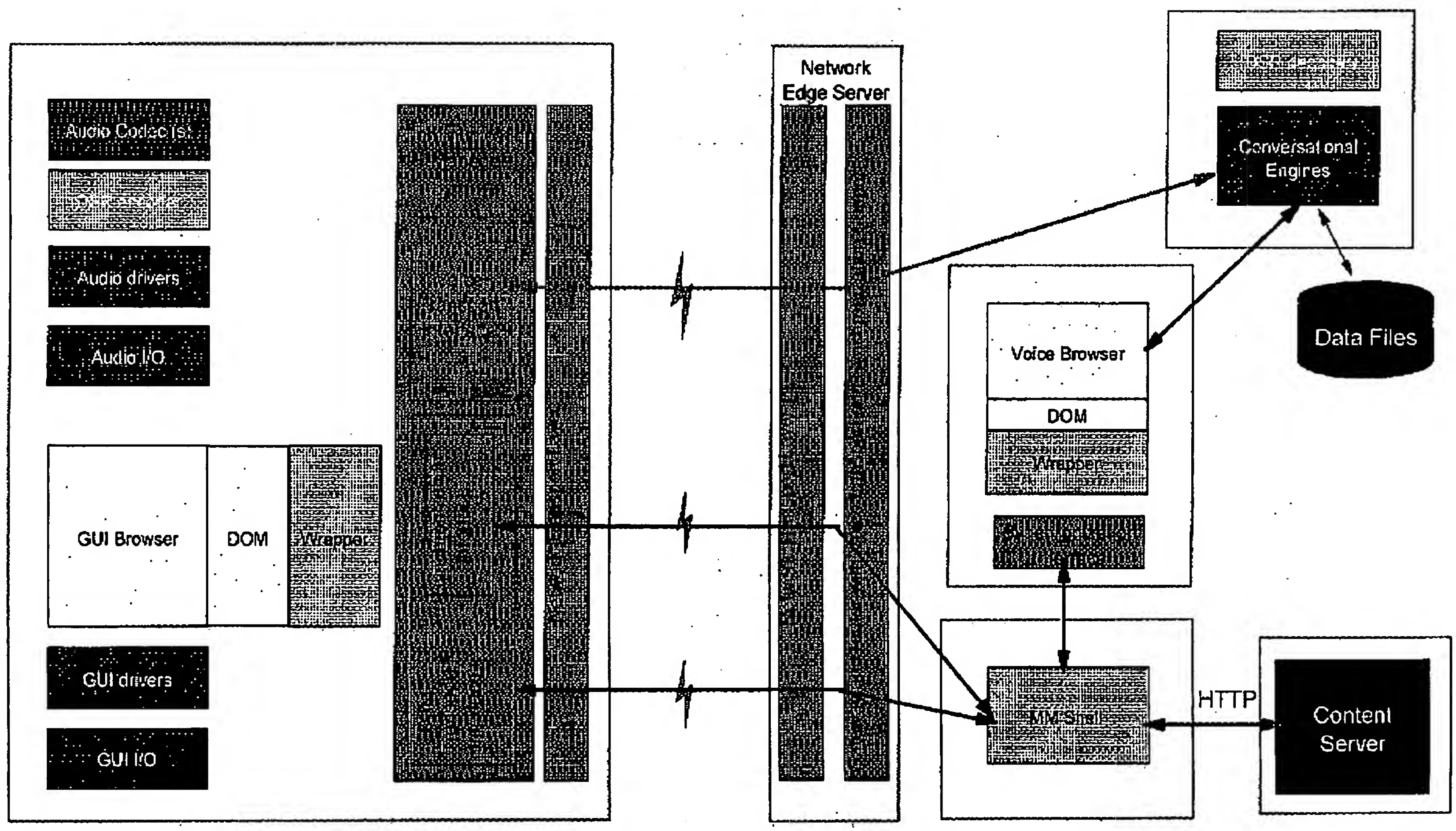
(See authoring discussion)

Legend:
Event Communication
- Next Step (not always present)
- Synchronization of the views

Target Multi-modal Architecture: Thin Client

- Recommended target aarchitecture for most 3G terminals (smart phones):
  - Enables small client foot-print
  - Synchronization and voice recognition / conversational functions are on the server-side

CLIENT-SIDE SERVER-SIDE

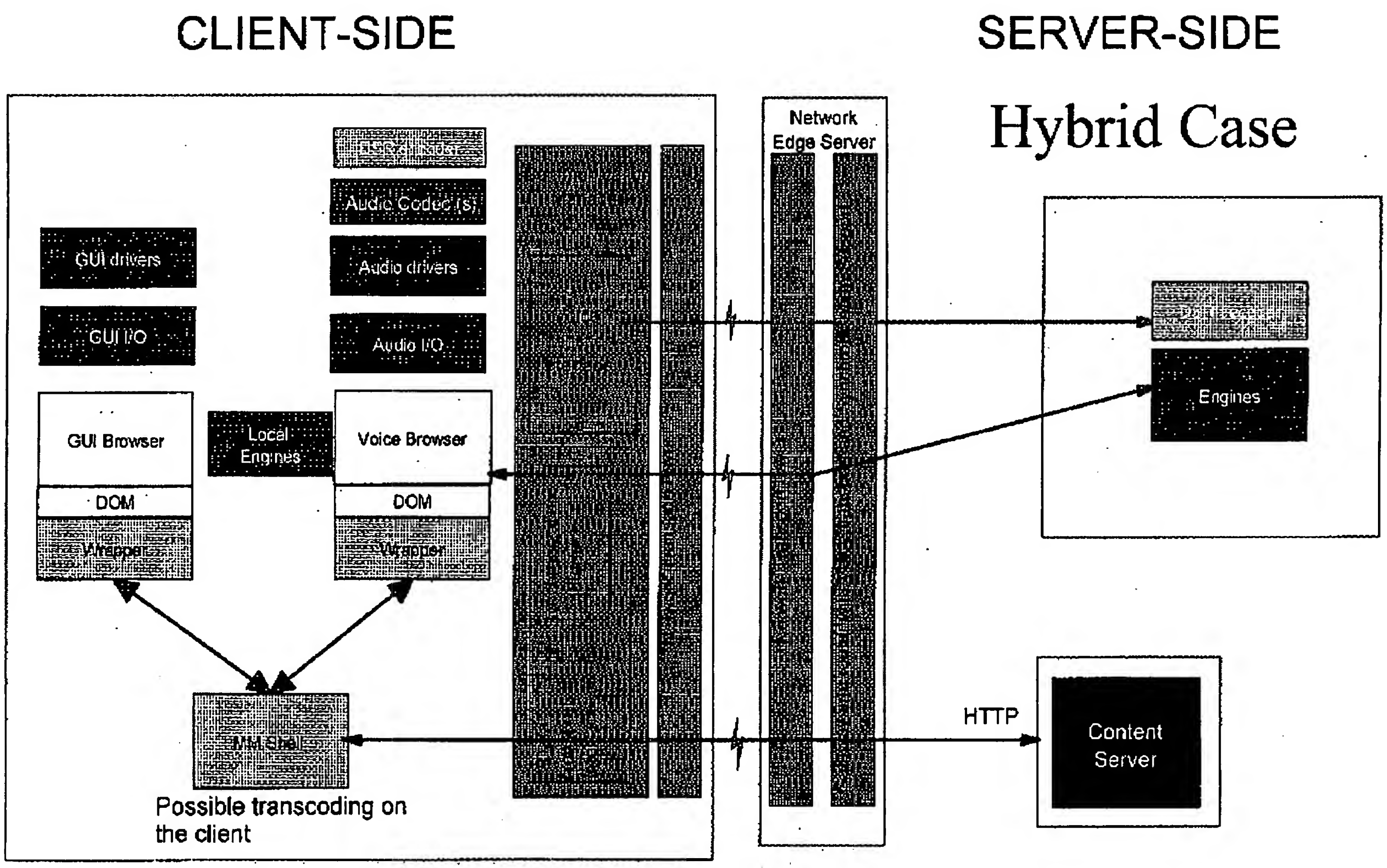


DSR is optional to improve performances of speech recognition. It can be done with existing codecs at the cost of accuracy drops

DSR - Distributed speech Recognition - See: T2-010627 (LS from SA-1: S1-010847)

Target Multi-modal Architecture: Fat Client

- Possible architecture with fatter terminals
- Requires resources to synchronize and for speech recognition / conversational engines
- Fat configuration support disconnected usage
- Hybrid case supports case where embedded dient side-speech recognition capabilities are too limited for the task

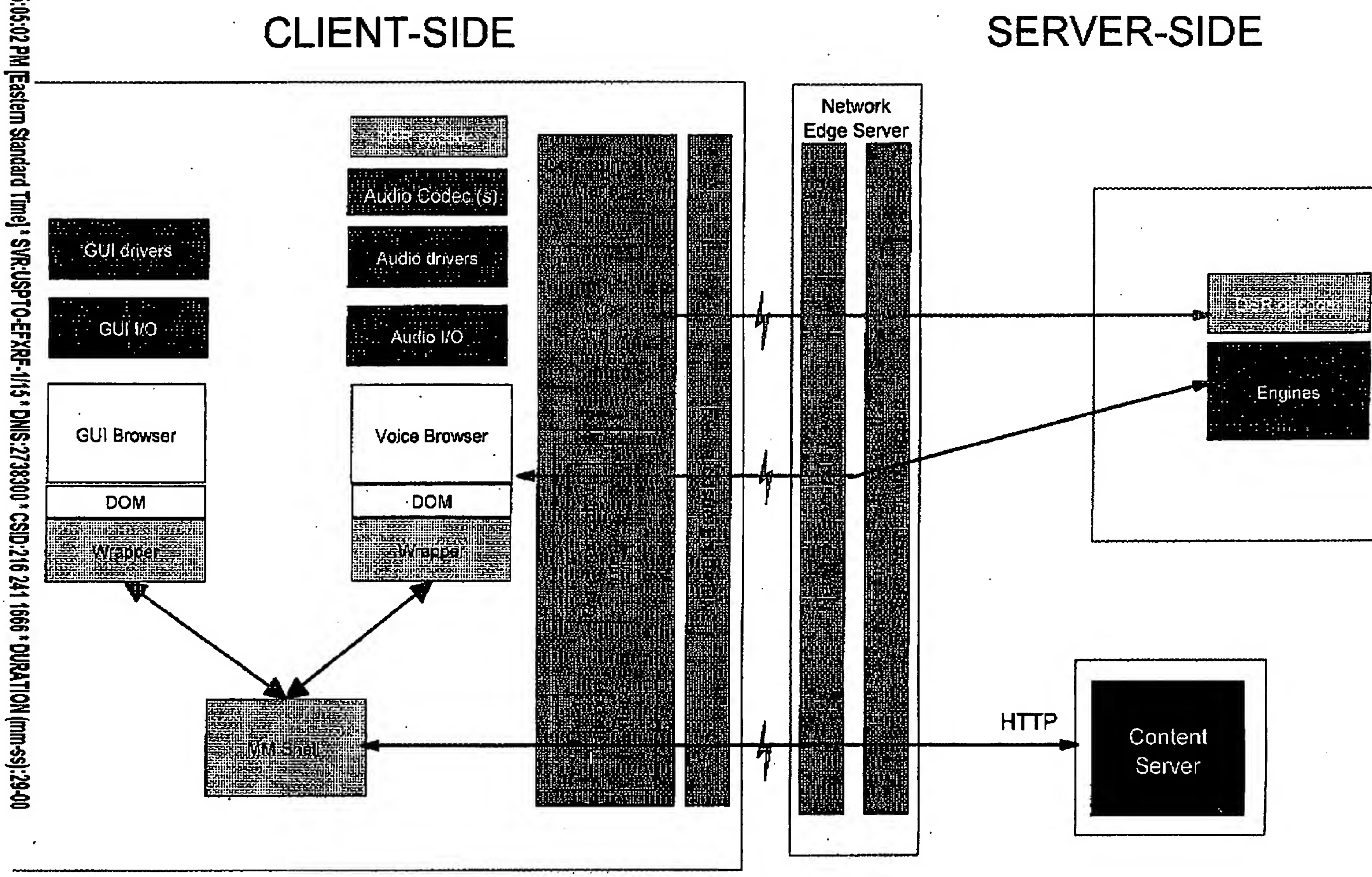


DSR is optional

DSR - Distributed speech Recognition - See: T2-010627 (LS from SA-1: S1-010847)

Variation of the fat client configuration - DSR and server side speech recognition

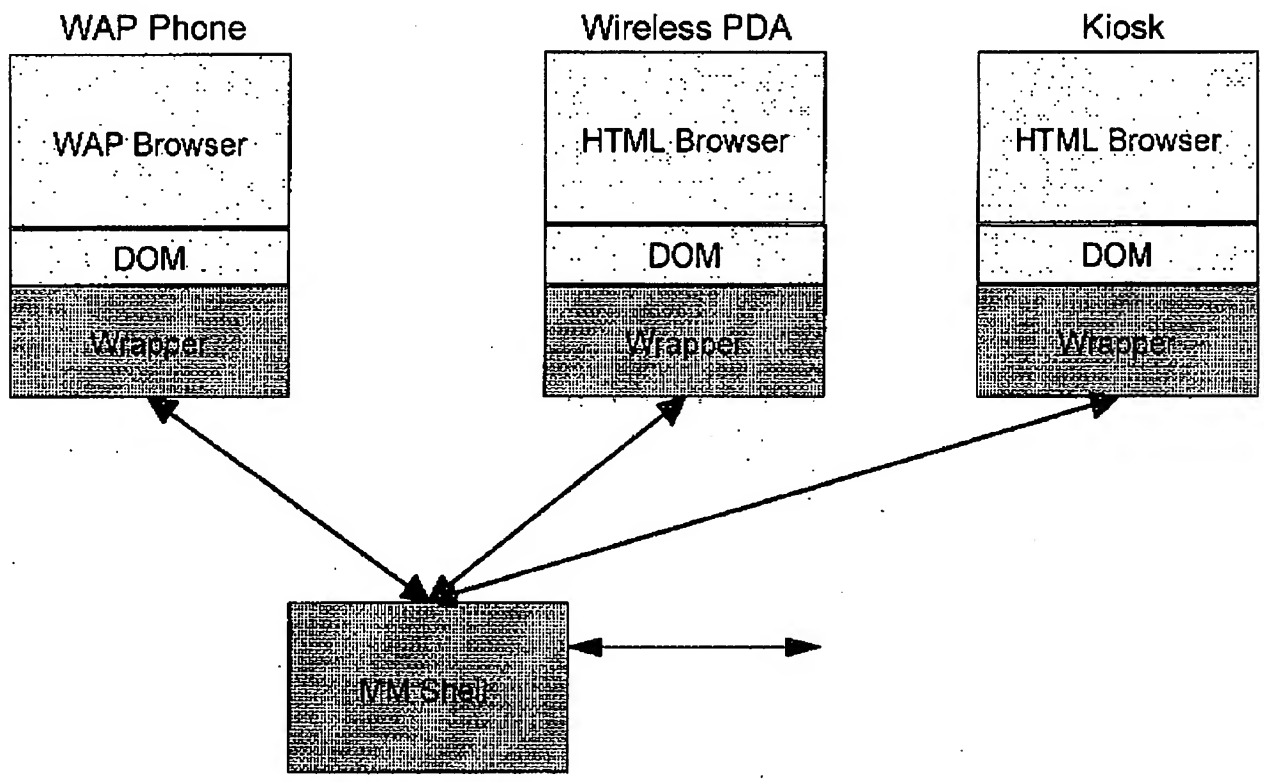
- This can address requirements to maintain the "context" on the client



DSR is optional

Multi-device Browser Configuration

- Recommended target aarchitecture for multi-device browsing
  - Related to UEsplit (3GPP Activity)



Each piece is distributable

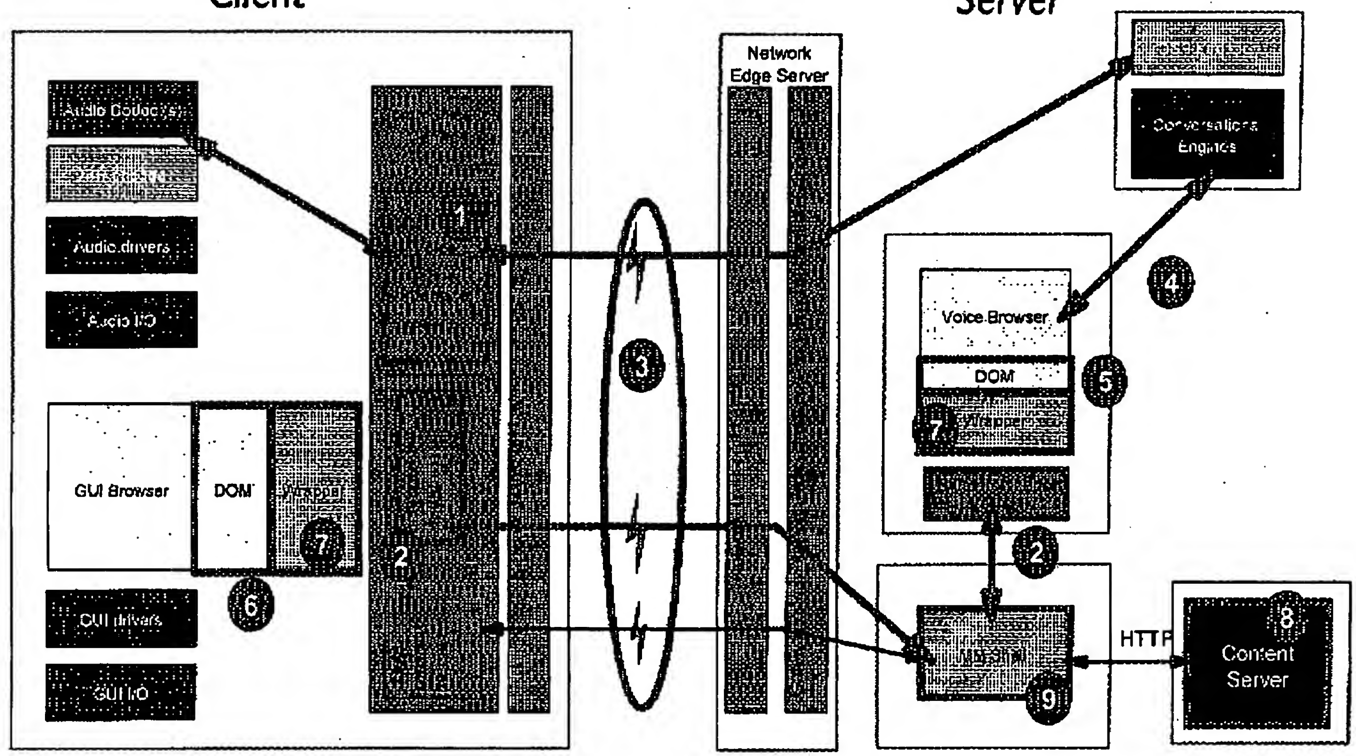
What is needed?

Infrastructure Requirements and current inhibitors

- **Client:**
  - DOM-L2 compliant browsers and Wrapper (or look alike or subset)
  - Support for synchronization protocols (e.g. SOAP)
    - SOAP (1.1) is currently defined by W3C as XML protocol
  - Support for Voice and Data (VoIP, DSR stack (SIP, SDP,SOAP, Payload),...)
  - Capabilities (audio sub-system; CPU / memory for Fat client configurations)
  - Channel / user descriptors: delivery context descriptors
  - Dynamic discosvery and bindings (later)
- **Network and gateways:**
  - Support voice and data (**DSR protocol stack** - T2-010627)
  - Support synchronization protocols (**SOAP over SIP**)
  - Support session / user information exchanges (Delivery context)
- **Server middleware:**
  - Supports:
    - voice and data (DSR protocol stack)
    - Synchronization protocols (SOAP)
    - Session / user management (delivery context)
    - Synchronization, state persistence
- **Authoring:**
  - Standards
  - Tools

These inhibitors should disappear in the next 2 to 5 years.

Protocols, Interfaces and Components to Standardize



Thin Client Configuration / Multi-device

1) **ETSI - STQ, IETF-AVT and 3GPP, ITU-SG16**
2) **W3C (XML Protocols** / MM), ETSI, **WAP Forum, 3GPP**
   W3C DI for delivery context
3) IETF, **3GPP**, WAP Forum
4) **ETSI - STQ**
5) **W3C Voice Activity**
6) **WAP Forum**, W3C, ETSI-STQ, 3GPP?
7) W3C MM WG, **WAP Forum** (WAE - Mobile DOM), 3GPP?
8) **W3C** (DI, XForms, MM, etc...), **WAP Forum**
9) W3C, WAP Forum

Fat Client Configuration:

5) **W3C Voice activity**
6) **WAP Forum**, W3C, ETSI-STQ, 3GPP?
7) W3C MM WG, **WAP Forum**, 3GPP?
8) **W3C** (DI, XForms, MM, etc...), **WAP Forum**
9) W3C, WAP Forum

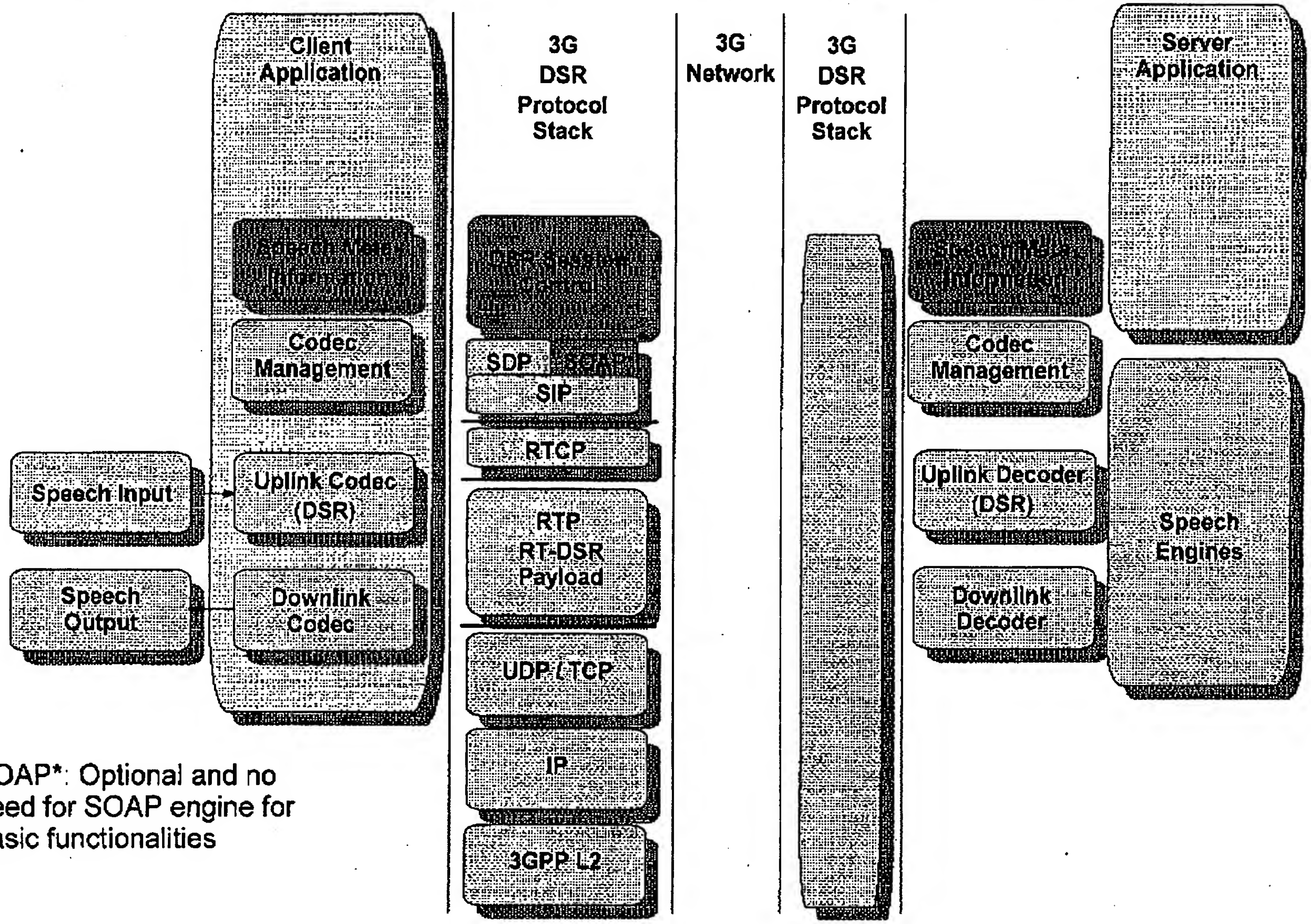
etc...

Conclusions

- Multi-modal and Multi-device browsing can accelerate the growth of wireless internet and m-Commerce
- Inhibitors will disappear in next 2 to 5 years
- Standards are key to eliminate the inhibitors and seed the market
- We have proposed a standard-based flexible, modular and extensible architecture and associated programming model
- Numerous items could be addressed by 3GPP:
  - Support for DSR and Multi-modal protocols stack (client, network, server and gateways):
    - **SOAP over SIP**
      - SOAP is currently defined by W3C as XML Protocol.
        - Currently 1.1 version exists with bidnings over HTTP for example
      - SOAP over SIP: to be done. Different proposals exist.
        - IBM has a simple implementation proposal
    - To support the stack will defacto enable multi-modal and multi-device deloyments when **User Agent** offers DOM L1/L2 appropriate interface (e.g. WAP)
  - Support for architecture, authoring and standardization elsewhere
  - Inclusion of compatibility requirements in current standardization activities
  - **Client components (DOM L1/L2, wrapper, SOAP support)**

Background Material

Distributed Speech Recognition - for 3GPP Release 5 - A first step - MM angle



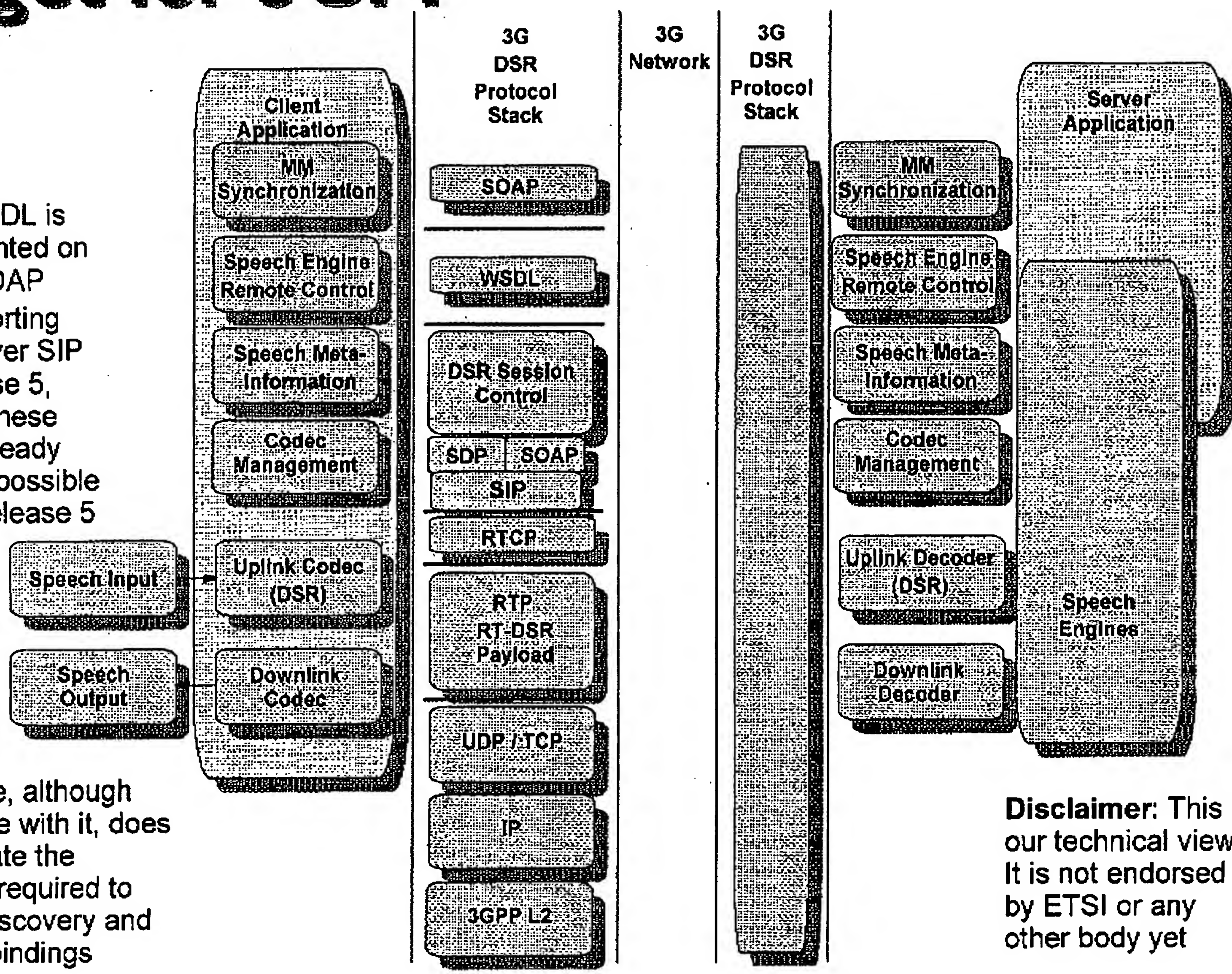
SOAP*: Optional and no need for SOAP engine for basic functionalities

See: T2-010627 (LS from SA-1: S1-010847) - In magenta: items under discussion at ETSI STQ DSR A&P: IBM Proposal to ETSI for 3GPP submission (not a ETSI endorsed item at this stage)

Distributed Speech Recognition and Multi-modal Protocol stack - Possible Target for 3GPP

Note WSDL is implemented on top of SOAP

By supporting SOAP over SIP in Release 5, most of these would already become possible within Release 5



This figure, although compatible with it, does not illustrate the protocols required to support discovery and dynamic bindings

Disclaimer: This is our technical view. It is not endorsed by ETSI or any other body yet

STANFORD ENCYCLOPEDIA OF PHILOSOPHY

Modal Logic

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A modal is an expression (like 'necessarily' or 'possibly') that is used to qualify the truth of a judgement. Modal logic is, strictly speaking, the study of the deductive behavior of the expressions 'it is necessary that' and 'it is possible that'. However, the term 'modal logic' may be used more broadly for a family of related systems. These include logics for belief, for tense and other temporal expressions, for the deontic (moral) expressions such as 'it is obligatory that' and 'it is permitted that', and many others. An understanding of modal logic is particularly valuable in the formal analysis of philosophical argument, where expressions from the modal family are both common and confusing. Modal logic also has important applications in computer science.

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1. What is Modal Logic?

Narrowly construed, modal logic studies reasoning that involves the use of the expressions 'necessarily' and 'possibly'. However, the term 'modal logic' is used more broadly to cover a family of logics with similar rules and a variety of different symbols.

A list describing the best known of these logics follows.

Logic	Symbols	Expressions Symbolized
Modal Logic	$\Box$	It is necessary that ..
	$\Diamond$	It is possible that ..
Deontic Logic	O	It is obligatory that ..
	P	It is permitted that ..
	F	It is forbidden that ..
Temporal Logic	G	It will always be the case that ..
	F	It will be the case that ..
	H	It has always been the case that ..
	P	It was the case that..
Doxastic Logic	Bx	x believes that ..

2. Modal Logics

The most familiar logics in the modal family are constructed from a weak logic called K (after Saul Kripke). Under the narrow reading, modal logic concerns necessity and possibility. A variety of different systems may be developed for such logics using K as a foundation. The symbols of K include '$\sim$' for 'not', '$\rightarrow$' for 'if...then', and '$\Box$' for the modal operator 'it is necessary that'. (The connectives '&', '$\vee$', and '$\leftrightarrow$' may be defined from '$\sim$' and '$\rightarrow$' as is done in propositional logic.) K results from adding the following to the principles of propositional logic.

Necessitation Rule: If A is a theorem of K, then so is $\Box A$.

Distribution Axiom: $\Box(A\rightarrow B) \rightarrow (\Box A\rightarrow\Box B)$.

(In these principles we use 'A' and 'B' as metavariables ranging over formulas of the language.) According to the Necessitation Rule, any theorem of logic is necessary. The Distribution Axiom says that if it is necessary that if A then B, then if necessarily A then necessarily B.

The operator $\Diamond$ (for 'possibly') can be defined from $\Box$ by letting $\Diamond A = \sim\Box\sim A$. In K, the operators $\Box$ and $\Diamond$ behave very much like the quantifiers $\forall$ (all) and $\exists$ (some). For example, the definition of $\Diamond$ from $\Box$ mirrors the equivalence of $\forall xA$ with $\sim\exists x\sim A$ in predicate logic. Furthermore, $\Box(A\&B)$ entails $\Box A\&\Box B$ and vice versa; while $\Box A \vee \Box B$ entails $\Box(A \vee B)$, but *not* vice versa. This reflects the patterns exhibited by the universal quantifier: $\forall x(A\&B)$ entails $\forall xA\&\forall xB$ and vice versa, while $\forall xA \vee \forall xB$ entails $\forall x(A \vee B)$ but not vice versa. Similar parallels between $\Diamond$ and $\exists$ can be drawn. The basis for this correspondence between the modal operators and the quantifiers will emerge more clearly in the section on Possible Worlds Semantics.

The system K is too weak to provide an adequate account of necessity. The following axiom is not provable in K, but it is clearly desirable.

(M) $\Box A\rightarrow A$

(M) claims that whatever is necessary is the case. Notice that (M) would be incorrect were $\Box$ to be read 'it ought to be that', or 'it was the case that'. So the presence of axiom (M) distinguishes modal from other logics in the modal family. A basic modal logic M results from adding (M) to K. (Some authors call this system T.)

Many logicians believe that M is still too weak to correctly formalize the logic of necessity and possibility. They recommend further axioms to govern the iteration, or repetition of modal operators. Here are two of the most famous iteration axioms:

(4) $\Box A\rightarrow\Box\Box A$

(5) $\Diamond A\rightarrow\Box\Diamond A$

S4 is the system that results from adding (4) to M. Similarly S5 is M plus (5). In S4, the sentence $\Box\Box A$ is equivalent to $\Box A$. As a result, any string of boxes may be replaced by a single box, and the same goes for strings of diamonds. This amounts to the idea that iteration of the modal operators is superfluous. Saying that A is necessarily necessary is considered a uselessly long-winded way of saying that A is necessary. The system S5 has even stronger principles for simplifying strings of modal operators. In S4, a string of operators of *the same kind* can be replaced for that operator; in S5, strings containing both boxes and diamonds are equivalent to the last operator in the string. So, for example, saying that it is possible that A is necessary is the same as saying that A is necessary. A summary of these features of S4 and S5 follows.

S4: $\Box\Box...\Box = \Box$ and $\Diamond\Diamond...\Diamond = \Diamond$

S5: $00...\Box = \Box$ and $00...\Diamond = \Diamond$, where each 0 is either $\Box$ or $\Diamond$

One could engage in endless argument over the correctness or incorrectness of these and other iteration principles for $\Box$ and $\Diamond$. The controversy can be partly resolved by recognizing that the words 'necessarily' and 'possibly', have many different uses. So the acceptability of axioms for modal logic depends on which of these uses we have in mind. For this reason, there is no one modal logic, but rather a whole family of systems built around M. The relationship between these systems is diagrammed in Section 8, and their application to different uses of 'necessarily' and 'possibly' can be more deeply understood by studying their possible world semantics in Section 6.

The system B (for the logician Brouw .) is formed by adding axiom (B) to M.

(B) $A \to \Box\Diamond A$

It is interesting to note that S5 can be formulated equivalently by adding (B) to S4. The axiom (B) raises an important point about the interpretation of modal formulas. (B) says that if A is the case, then A is necessarily possible. One might argue that (B) should always be adopted in any modal logic, for surely if A is the case, then it is necessary that A is possible. However, there is a problem with this claim that can be exposed by noting that $\Diamond\Box A \to A$ is provable from (B). So $\Diamond\Box A \to A$ should be acceptable if (B) is. However, $\Diamond\Box A \to A$ says that if A is possibly necessary, then A is the case, and this is far from obvious. Why does (B) seem obvious, while one of the things it entails seems not obvious at all? The answer is that there is a dangerous ambiguity in the English interpretation of $A \to \Box\Diamond A$. We often use the expression 'If A then necessarily B' to express that the conditional 'if A then B' is necessary. This interpretation corresponds to $\Box(A \to B)$. On other occasions, we mean that if A, then B is necessary: $A \to \Box B$. In English, 'necessarily' is an adverb, and since adverbs are usually placed near verbs, we have no natural way to indicate whether the modal operator applies to the whole conditional, or to its consequent. For these reasons, there is a tendency to confuse (B): $A \to \Box\Diamond A$ with $\Box(A \to \Diamond A)$. But $\Box(A \to \Diamond A)$ is not the same as (B), for $\Box(A \to \Diamond A)$ is already a theorem of M, and (B) is not. One must take special care that our positive reaction to $\Box(A \to \Diamond A)$ does not infect our evaluation of (B). One simple way to protect ourselves is to formulate B in an equivalent way using the axiom: $\Diamond\Box A \to A$, where these ambiguities of scope do not arise.

3. Deontic Logics

Deontic logics introduce the primitive symbol O for 'it is obligatory that', from which symbols P for 'it is permitted that' and F for 'it is forbidden that' are defined: $PA = \sim O \sim A$ and $FA = O \sim A$. The deontic analog of the modal axiom (M): $OA \to A$ is clearly not appropriate for deontic logic. (Unfortunately, what ought to be is not always the case.) However, a basic system D of deontic logic can be constructed by adding the weaker axiom (D) to K.

(D) $OA \to PA$

Axiom (D) guarantees the consistency of the system of obligations by insisting that when A is obligatory, A is permissible. A system which obligates us to bring about A, but doesn't permit us to do so, puts us in an inescapable bind. Although some will argue that such conflicts of obligation are at least possible, most deontic logicians accept (D).

$O(OA \to A)$ is another deontic axiom that seems desirable. Although it is wrong to say that if A is obligatory then A is the case ($OA \to A$), still, this conditional *ought* to be the case. So some deontic logicians believe that D needs to be supplemented with $O(OA \to A)$ as well.

Controversy about iteration (repetition) of operators arises again in deontic logic. In some conceptions of obligation, OOA just amounts to OA. 'It ought to be that it ought to be' is treated as a sort of stuttering; the extra 'ought's do not add anything new. So axioms are added to guarantee the equivalence of OOA and OA. The more general iteration policy embodied in S5 may also be adopted. However, there are conceptions of obligation where distinction between OA and OOA is preserved. The idea is that there are genuine differences between the obligations we *actually* have and the obligations we *should* adopt. So, for example, 'it ought to be that it ought to be that A' commands adoption of some obligation which may not actually be in place, with the result that OOA can be true even when OA is false.

4. Temporal Logics

In temporal logic (also known as tense logic), there are two basic operators, G for the future, and H for the past. G is read 'it always will be that' and the defined operator F (read 'it will be the case that'), can be introduced by $FA = \sim G \sim A$. Similarly H is read: 'it always was that' and P (for 'it was the case that') is defined by $PA = \sim H \sim A$. A basic system of temporal logic called Kt results from adopting the principles of K for both G and H, along with two axioms to govern the interaction between the past and future operators:

"Necessitation" Rules: If A is a theorem then so are GA and HA.

Distribution Axioms: $G(A \to B) \to (GA \to GB)$ and $H(A \to B) \to (HA \to HB)$

Interaction Axioms: $A \to GPA$ and $A \to HFA$

The interaction axioms raise questions concerning asymmetries between the past and the future. A standard intuition is that the

past is fixed, while the future is still open. The first interaction axiom (A→GPA) conforms to this intuition in reporting that what is the case (A), will at all future times, be in the past (GPA). However A→HFA may appear to have unacceptably deterministic overtones, for it claims, apparently, that what is true now (A) has always been such that it will occur in the future (HFA). However, possible world semantics for temporal logic reveals that this worry results from a simple confusion, and that the two interaction axioms are equally acceptable.

Note that the characteristic axiom of modal logic, (M): □A→A, is not acceptable for either H or G, since A does not follow from 'it always was the case that A', nor from 'it always will be the case that A'. However, it is acceptable in a closely related temporal logic where G is read 'it is and always will be', and H is read 'it is and always was'.

Depending on which assumptions one makes about the structure of time, further axioms must be added to temporal logics. A list of axioms commonly adopted in temporal logics follows. An account of how they depend on the structure of time will be found in the section Possible Worlds Semantics.

GA→GGA and HA→HHA

GGA→GA and HHA→HA

GA→FA and HA→PA

It is interesting to note that certain combinations of past tense and futuere tense operators may be used to express complex tenses in English. For example, FPA, corresponds to sentence A in the future perfect tense, (as in '20 seconds from now the light will have changed'). Similarly, PPA expresses the past perfect tense.

For a more detailed discussion of temporal logic, see the entry on temporal logic.

5. Conditional Logics

The founder of modal logic, C. I. Lewis, defined a series of modal logics which did not have □ as a primitive symbol. Lewis was concerned to develop of logic of conditionals that was free of the so called Paradoxes of Material Implication, namely the classical theorems A→(~A→B) and B→(A→B). He introduced the symbol ⥽ for "strict implication" and developed logics where neither A ⥽ (~A ⥽ B) nor B ⥽ (A ⥽ B) is provable. The modern practice has been to define A ⥽ B by □(A→B), and use modal logics governing □ to obtain similar results. However, the provability of such formulas as (A&~A) ⥽ B in such logics seems at odds with concern for the paradoxes. Anderson and Belnap (1975) have developed systems R (for Relevance Logic) and E (for Entailment) which are designed to overcome such difficulties. These systems require revision of the standard systems of propositional logic. (For a more detailed discussion of relevance logic, see the entry on relevance logic.)

David Lewis (1973) has developed special conditional logics to handle counterfactual expressions, that is, expressions of the form 'if A *were* to happen then B *would* happen'. (Kvart (1980) is another good source on the topic.) Counterfactual logics differ from those based on strict implication because the former reject while the latter accept contraposition.

6. Possible Worlds Semantics

The purpose of logic is to characterize the difference between valid and invalid arguments. A logical system for a language is a set of axioms and rules designed to prove *exactly* the valid arguments statable in the language. Creating such a logic may be a difficult task. The logican must make sure that the system is *sound*, i.e. that every argument proven using the rules and axioms is in fact valid. Furthermore, the system should be *complete*, meaning that every valid argument has a proof in the system. Demonstrating soundness and completeness of formal systems is a logician's central concern.

Such a demonstration cannot get underway until the concept of validity is defined rigorously. Formal semantics for a logic provides a definition of validity by characterizing the truth behavior of the sentences of the system. In propositional logic, validity can be defined using truth tables. A valid argument is simply one where every truth table row that makes its premises true also makes its conclusion true. However truth tables cannot be used to provide an account of validity in modal logics because there are no truth tables for expressions such as 'it is necessary that', 'it is obligatory that', and the like. (The problem is that the truth value of A does not determine the truth value for □A. For example, when A is 'Dogs are dogs', □A is true, but when A is 'Dogs are pets', □A is false. Nevertheless, semantics for modal logics can be defined by introducing possible worlds. We will illustrate possible worlds semantics for a logic of necessity containing the symbols ~, →, and □. Then we will explain how the same strategy may be adapted to other logics in the modal family.

In propositional logic, a valuation of the atomic sentences (or row of a truth table) assigns a truth value (T or F) to each propositional variable p. Then the truth values of the complex sentences is calculated with truth tables. In modal semantics, a set W of possible worlds is introduced. A valuation then gives a truth value to each propositional variable *for each of the possible worlds* in W. This means that value assigned to p for world w may differ from the value assigned to p for another world w′ .

The truth value of the atomic sentence p at world w given by the valuation v may be written v(p, w). Given this notation, the truth values (T for true, F for false) of complex sentences of modal logic for a given valuation v (and member w of the set of worlds W) may be defined by the following truth clauses. ('iff' abbreviates 'if and only if'.)

(~) v(~A, w)=T iff v(A, w)=F.

(→) v(A→B, w)=T iff v(A, w)=F or v(B, w)=T.

(5) v(□A, w)=T iff for every world w′ in W, v(A, w′)=T.

Clauses (~) and (→) simply describe the standard truth table behavior for negation and material implication respectively. According to (5), □A is true (at a world w) exactly when A is true in *all* possible worlds. Given the definition of ◊, (namely, ◊A = ~□~A) the truth condition (5) insures that ◊A is true just in case A is true in *some* possible world. Since the truth clauses for □ and ◊ involve the quantifiers 'all' and 'some' (respectively), the parallels in logical behavior between □ and ∀x, and between ◊ and ∃x noted in section 2 will be expected.

Clauses (~), (→), and (5) allow us to calculate the truth value of any sentence at any world on a given valuation. A definition of validity is now just around the corner. An argument is *5-valid for a given set W* (of possible worlds) if and only if every valuation of the atomic sentences that assigns the premises T at a world in W also assigns the conclusion T at the same world. An argument is said to be *5-valid* iff it is valid for every non empty set of W of possible worlds.

It has been shown that S5 is sound and complete for 5-validity (hence our use of the symbol '5'). The 5-valid arguments are exactly the arguments provable in S5. This result suggests that S5 is the correct way to formulate a logic of necessity.

However, S5 is not a reasonable logic for all members of the modal family. In deontic logic, temporal logic, and others, the analog of the truth condition (5) is clearly not appropriate; furthermore there are even conceptions of necessity where (5) should be rejected as well. The point is easiest to see in the case of temporal logic. Here, the members of W are moments of time, or worlds "frozen", as it were, at an instant. For simplicity let us consider a *future* temporal logic, a logic where □A reads: 'it *will* always be the case that'. (We formulate the system using □ rather than the traditional G so that the connections with other modal logics will be easier to appreciate.) The correct clause for □ should say that □A is true at time w iff A is true at all times *in the future of w*. To restrict attention to the future, the relation R (for 'eaRlier than') needs to be introduced. Then the correct clause can be formulated as follows.

(K) v(□A, w)=T iff for every w′ , if wRw′ , then v(A, w′)=T.

This says that □A is true at w just in case A is true at all times *after* w.

Validity for this brand of temporal logic can now be defined. A *frame* <W, R> is a pair consisting of a non-empty set W (of worlds) and a binary relation R on W. A *model* <F, v> consists of a frame F, and a valuation v that assigns truth values to each atomic sentence at each world in W. Given a model, the values of all complex sentences can be determined using (~), (→), and (K). An argument is K-valid just in case any model whose valuation assigns the premises T at a world also assigns the conclusion T at the same world. As the reader may have guessed from our use of 'K', it has been shown that the simplest modal logic K is both sound and complete for K-validity.

7. Modal Axioms and Conditions on Frames

One might assume from this discussion that K is the correct logic when □ is read 'it will always be the case that'. However, there are reasons for thinking that K is too weak. One obvious logical feature of the relation R (earlier than) is transitivity. If wRv (w is earlier than v) and vRu (v is earlier than u), then it follows that wRu (w is earlier than u). So let us define a new kind of validity that corresponds to this condition on R. Let a 4-model be any model whose frame <W, R> is such that R is a transitive relation on W. Then an argument is 4-valid iff any 4-model whose valuation assigns T to the premises at a world also assigns T to the conclusion at the same world. We use '4' to describe such a transitive model because the logic which is adequate (both sound and complete) for 4-validity is K4, the logic which results from adding the axiom (4): □A→□□A to K.

Transitivity is not the only property which we might want to require of the frame <W, R> if R is to be read 'earlier than' and W

is a set of moments. One condition (which is only mildly controversial) is that there is no last moment of time, i.e. that for every world w there is some world v such that wRv. This condition on frames is called *seriality*. Seriality corresponds to the axiom (D): $\Box A \rightarrow \Diamond A$, in the same way that transitivity corresponds to (4). A D-model is a K-model with a serial frame. From the concept of a D-model the corresponding notion of D-validity can be defined just as we did in the case of 4-validity. As you probably guessed, the system that is adequate with respect to D-validity is KD, or K plus (D). Not only that, but the system KD4 (that is K plus (4) and (D)) is adequate with respect to D4-validity, where a D4-model is one where <W, R> is *both* serial and transitive.

Another property which we might want for the relation 'earlier than' is density, the condition which says that between any two times we can always find another. Density would be false if time were atomic, i.e. if there were intervals of time which could not be broken down into any smaller parts. Density corresponds to the axiom (C4): $\Box\Box A \rightarrow \Box A$, the converse of (4), so for example, the system KC4, which is K plus (C4) is adequate with respect to models where the frame <W, R> is dense, and KDC4, adequate with respect to models whose frames are serial and dense, and so on.

Each of the modal logic axioms we have discussed corresponds to a condition on frames in the same way. The relationship between conditions on frames and corresponding axioms is one of the central topics in the study of modal logics. Once an interpretation of the intensional operator $\Box$ has been decided on, the appropriate conditions on R can be determined to fix the corresponding notion of validity. This, in turn, allows us to select the right set of axioms for that logic.

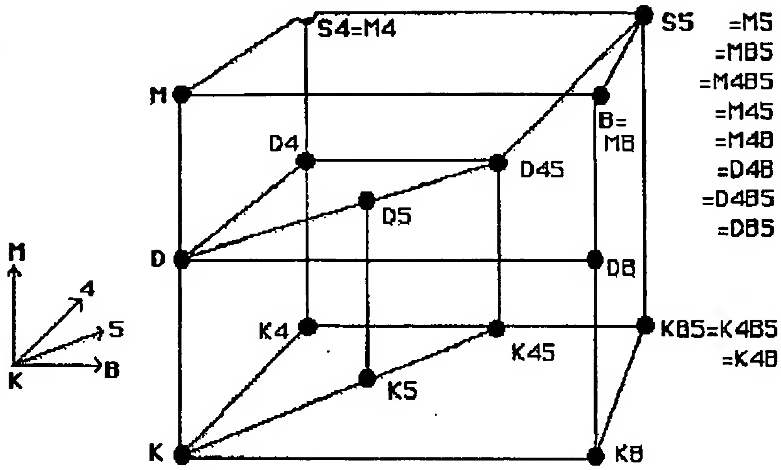
For example, consider a deontic logic, where $\Box$ is read 'it is obligatory that'. Here the truth of $\Box A$ does not demand the truth of A in *every* possible world, but only in a subset of those worlds where people do what they ought. So we will want to introduce a relation R for for this kind of logic as well, and use the truth clause (K) to evaluate $\Box A$ at a world. However, in this case, R is not earlier than. Instead wRw′ holds just in case world w′ is a morally acceptable variant of w, i.e. a world that our actions can bring about which satisfies what is morally correct, or right, or just. Under such a reading, it should be clear that the relevant frames should obey seriality, the condition that requires that each possible world have a morally acceptable variant. The analysis of the properties desired for R makes it clear that a basic deontic logic can be formulated by adding the axiom (D) and to K.

Even in modal logic, one may wish to restrict the range of possible worlds which are relevant in determining whether $\Box A$ is true at a given world. For example, I might say that it is necessary for me to pay my bills, even though I know full well that there is a possible world where I fail to pay them. In ordinary speech, the claim that A is necessary does not require the truth of A in *all* possible worlds, but rather only in a certain class of worlds which I have in mind (for example, worlds where I avoid penalties for failure to pay). In order to provide a generic treatment of necessity, we must say that $\Box A$ is true in w iff A is true in all worlds *that are related* to w in the right way. So for an operator $\Box$ interpreted as necessity, we introduce a corresponding relation R on the set of possible worlds W, traditionally called the accessibility relation. The accessibility relation R holds between worlds w and w′ iff w′ is possible given the facts of w. Under this reading for R, it should be clear that frames for modal logic should be reflexive. It follows that modal logics should be founded on M, the system that results from adding (M) to K. Depending on exactly how the accessibility relation is understood, symmetry and transitivity may also be desired.

A list of some of the more commonly discussed conditions on frames and their corresponding axioms along with a map showing the relationship between the various modal logics can be found in the next section.

8. Map of the Relationships Between Modal Logics

The following diagram shows the relationships between the best known modal logics, namely logics that can be formed by adding a selection of the axioms (D), (M), (4), (B) and (5) to K. A list of these (and other) axioms along with their corresponding frame conditions can be found below the diagram.



In this chart, systems are given by the list of their axioms. So, for example M4B is the result of adding (M) (4) and (B) to K. In boldface, we have indicated traditional names of some systems. When system S appears below and/or to the left of S′ connected by a line, then S′ is an extension of S. This means that every argument provable in S is provable in S′, but S is weaker than S′, i.e. not all arguments provable in S′ are provable in S.

The following list indicates axioms, their names, and the corresponding conditions on the accessibility relation R, for axioms so far discussed in this encyclopedia entry.

Axiom Name	Axiom	Condition on Frames	R is...
(D)	$\Box A \to \Diamond A$	$\exists u\, wRu$	Serial
(M)	$\Box A \to A$	wRw	Reflexive
(4)	$\Box A \to \Box\Box A$	$(wRv \& vRu) \Rightarrow wRu$	Transitive
(B)	$A \to \Box\Diamond A$	$wRv \Rightarrow vRw$	Symmetric
(5)	$\Diamond A \to \Box\Diamond A$	$(wRv \& wRu) \Rightarrow vRu$	Euclidean
(CD)	$\Diamond A \to \Box A$	$(wRv \& wRu) \Rightarrow v=u$	Unique
($\Box$M)	$\Box(\Box A \to A)$	$wRv \Rightarrow vRv$	Shift Reflexive
(C4)	$\Box\Box A \to \Box A$	$wRv \Rightarrow \exists u(wRu \& uRv)$	Dense
(C)	$\Diamond\Box A \to \Box\Diamond A$	$wRv \& wRx \Rightarrow \exists u(vRu \& xRu)$	Convergent

In the list of conditions on frames, the variables 'w', 'v', 'u', 'x' and the quantifier '$\exists u$' are understood to range over W. '&' abbreviates 'and' and '$\Rightarrow$' abbreviates 'if...then'.

9. The General Axiom

The correspondence between axioms and conditions on frames may seem something of a mystery. A beautiful result of Lemmon and Scott (1977) goes a long way towards explaining those relationships. Their theorem concerned axioms which have the following form:

(G) $\Diamond^h\Box^i A \to \Box^j\Diamond^k A$

We use the notation '$\Diamond^n$' to represent n diamonds in a row, so, for example, '$\Diamond^3$' abbreviates a string of three diamonds: '$\Diamond\Diamond\Diamond$'. Similarly '$\Box^n$' represents a string of n boxes. When the values of h, i, j, and k are all 1, we have axiom (C):

(C) $\Diamond\Box A \to \Box\Diamond A = \Diamond^1\Box^1 A \to \Box^1\Diamond^1 A$

The axiom (B) results from setting h and k to 0, and letting j and k be 1:

(B) $A \to \Box\Diamond A = \Diamond^0\Box^0 A \to \Box^1\Diamond^1 A$

To obtain (4), we may set h and k to 0, set i to 1 and j to 2:

(4) $\Box A \to \Box\Box A = \Diamond^0\Box^1 A \to \Box^2\Diamond^0 A$

Many (but not all) axioms of modal logic can be obtained by setting the right values for the parameters in (G)

Our next task will be to give the condition on frames which corresponds to (G) for a given selection of values for h, i, j, and k. In order to do so, we will need a definition. The composition of two relations R and R′ is a new relation $R\circ R'$ which is defined as follows:

$wR\circ R'\ v$ iff for some u, wRu and uR′ v.

For example, if R is the relation of being a brother, and R′ is the relation of being a parent then $R\circ R'$ is the relation of being an uncle, (because w is the uncle of v iff for some person u, both w is the brother of u and u is the parent of v). A relation may be composed with itself. For example, when R is the relation of being a parent, then $R\circ R$ is the relation of being a grandparent, and $R\circ R\circ R$ is the relation of being a great-grandparent. It will be useful to write 'R^n', for the result of composing R with itself n times. So R^2 is $R\circ R$, and R^4 is $R\circ R\circ R\circ R$. We will let R^1 be R, and R^0 will be the identity relation, i.e. wR^0v iff w=v.

We may now state the Scott-Lemmon result. It is that the condition on frames which corresponds exactly to any axiom of the shape (G) is the following.

(hijk-Convergence) $wR^hv\ \&\ wR^ju \Rightarrow \exists x\,(vR^ix\ \&\ uR^kx)$

It is interesting to see how the familiar conditions on R result from setting the values for h, i, j, and k according to the values in the corresponding axiom. For example, consider (5). In this case i=0, and h=j=k=1. So the corresponding condition is

$wRv\ \&\ wRu \Rightarrow \exists x\,(vR^0x\ \&\ uRx)$.

We have explained that R^0 is the identity relation. So if vR^0x then v=x. But $\exists x\,(v=x\ \&\ uRx)$, is equivalent to uRv, and so the Euclidean condition is obtained:

$(wRv\ \&\ wRu) \Rightarrow uRv$.

In the case of axiom (4), h=0, i=1, j=2 and k=0. So the corresponding condition on frames is

$(w=v\ \&\ wR^2u) \Rightarrow \exists x\,(vRx\ \&\ u=x)$.

Resolving the identities this amounts to:

$vR^2u \Rightarrow vRu$.

By the definition of R^2, vR^2u iff $\exists x(vRx\ \&\ xRu)$, so this comes to:

$\exists x(vRx\ \&\ xRu) \Rightarrow vRu$,

which by predicate logic, is equivalent to transitivity.

$vRx\ \&\ xRu \Rightarrow vRu$.

The reader may find it a pleasant exercise to see how the corresponding conditions fall out of hijk-Convergence when the values of the parameters h, i, j, and k are set by other axioms.

The Scott-Lemmon results provides a quick method for establishing results about the relationship between axioms and their corresponding frame conditions. Since they showed the adequacy of any logic that extends K with a selection of axioms of the

form (G) with respect to models that satisfy the corresponding set of frame conditions, they provided "wholesale" adequacy proofs for the majority of systems in the modal family. Sahlqvist (1975) has discovered important generalizations of the Scott-Lemmon result covering a much wider range of axiom types.

10. Provability Logics

Modal logic has been useful in clarifying our understanding of central results concerning provability in the foundations of mathematics (Boolos, 1993). Provability logics are systems where the propositional variables p, q, r, etc. range over formulas of some mathematical system, for example Peano's system PA for arithmetic. (The system chosen for mathematics might vary, but assume it is PA for this discussion.) Gödel showed that arithmetic has strong expressive powers. Using code numbers for arithmetic sentences, he was able to demonstrate a correspondence between sentences of mathematics and facts about which sentences are and are not provable in PA. For example, he showed there there is a sentence C that is true just in case no contradiction is provable in PA and there is a sentence G (the famous Gödel sentence) that is true just in case it is not provable in PA.

In provability logics, $\Box p$ is interpreted as a formula (of arithmetic) that expresses that what p denotes is provable in PA. Using this notation, sentences of provability logic express facts about provability. Suppose that $\bot$ is a constant of provability logic denoting a contradiction. Then $\sim\Box\bot$ says that PA is consistent and $\Box A\rightarrow A$ says that PA is sound in the sense that when it proves A, A is indeed true. Furthermore, the box may be iterated. So, for example, $\Box\sim\Box\bot$ makes the dubious claim that PA is able to prove its own consistency, and $\sim\Box\bot\rightarrow\sim\Box\sim\Box\bot$ asserts (correctly as Gödel proved) that if PA is consistent then PA is unable to prove its own consistency.

Although provability logics form a family of related systems, the system GL is by far the best known. It results from adding the following axiom to K:

(GL) $\Box(\Box A\rightarrow A)\rightarrow\Box A$

The axiom (4): $\Box A\rightarrow\Box\Box A$ is provable in GL, so GL is actually a strengthening of K4. However, axioms such as (M): $\Box A\rightarrow A$, and even the weaker (D): $\Box A\rightarrow\Diamond A$ are not available (nor desirable) in GL. In provability logic, provability is not to be treated as a brand of necessity. The reason is that when p is provable in an arbitrary system S for mathematics, it does not follow that p is true, since S may be unsound. Furthermore, if p is provable in S ($\Box p$) it need not even follow that $\sim p$ lacks a proof ($\sim\Box\sim p = \Diamond p$). S might be inconsistent and so prove both p and $\sim p$.

Axiom (GL) captures the content of Loeb's Theorem, an important result in the foundations of arithmetic. $\Box A\rightarrow A$ says that PA is sound for A, i.e. that if A were proven, A would be true. (Such a claim might not be secure for an arbitrarily selected sytem S, since A might be provable in S and false.) (GL) claims that if PA manages to prove the sentence that claims soundness for a given sentence A, then A is already provable in PA. Loeb's Theorem reports a kind of modesty on PA's part (Boolos, 1993, p. 55). PA never insists (proves) that a proof of A entails A's truth, unless it already has a proof of A to back up that claim.

It has been shown that GL is adequate for provability in the following sense. Let a sentence of GL be *always provable* exactly when the sentence of arithmetic it denotes is provable no matter how its variables are assigned values to sentences of PA. Then the provable sentences of GL are exactly the sentences that are always provable. This adequacy result has been extremely useful, since general questions concerning provability in PA can be transformed into easier questions about what can be demonstrated in GL.

GL can also be outfitted with a possible world semantics for which it is sound and complete. A corresponding condition on frames for GL-validity is that the frame be transitive, finite and irreflexive.

11. Quantifiers in Modal Logic

It would seem to be a simple matter to outfit a modal logic with the quantifiers $\forall$ (all) and $\exists$ (some). One would simply add the standard (or classical) rules for quantifiers to the principles of whichever propositional modal logic one chooses. However, systems of this kind create problems which have motivated some logicians to abandon classical quantifier rules in favor of the weaker rules of free logic (Garson, 1984). The controversy over whether classical principles should be adopted continues today.

The main points of disagreement can be traced back to decisions about how to handle the domain of quantification. The simplest alternative, the fixed-domain (sometimes called the possibilist) approach, assumes a single domain of quantification that contains all the possible objects. On the other hand, the world-relative (or actualist) interpretation, assumes that the domain of quantification changes from world to world, and contains only the objects that actually exist in a given world.

The fixed-domain approach requires ... major adjustments to the classical machinery for the quantifiers. Modal logics that are adequate for fixed domain semantics can usually be axiomatized by adding principles of a propositional modal logic to classical quantifier rules together with the Barcan Formula (BF) (Barcan 1946). (For an account of some interesting exceptions see Cresswell (1995)).

(BF) $\forall x\Box A\rightarrow\Box\forall xA$.

The fixed-domain interpretation has advantages of simplicity and familiarity, but it does not provide a direct account of the semantics of certain quantifier expressions of natural language. We do not think that 'Some man exists who signed the Declaration of Independence' is true, at least not if we read 'exists' in the present tense. Nevertheless, this sentence was true in 1777, which shows that the domain for the natural language expression 'some man exists who' changes to reflect which men exist at different times. A related problem is that on the fixed-domain interpretation, the sentence $\forall y\Box\exists x(x=y)$ is valid. However, assuming that $\exists x(x=y)$ is read: y exists, $\forall y\Box\exists x(x=y)$ says that everything exists necessarily. However, it seems a fundamental feature of common ideas about modality that the existence of many things is contingent, and that different objects exist in different possible worlds.

The defender of the fixed-domain interpretation may respond to these objections by insisting that on his (her) reading of the quantifiers, the domain of quantification contains *all* possible objects, not just the objects that happen to exist at a given world. So the theorem $\forall y\Box\exists x(x=y)$ makes the innocuous claim that every *possible* object is necessarily found in the domain of all possible objects. Furthermore, those quantifier expressions of natural language whose domain is world (or time) dependent can be expressed using the fixed-domain quantifier $\exists x$ and a predicate letter E with the reading 'actually exists'. For example, instead of translating 'Some Man exists who Signed the Declaration of Independence' by

$\exists x(Mx\&Sx)$,

the defender of fixed domains may write:

$\exists x(Ex\&Mx\&Sx)$,

thus ensuring the translation is counted false at the present time. Cresswell (1991) makes the interesting observation that world-relative quantification has limited expressive power relative to fixed-domain quantification. World-relative quantification can be defined with fixed domain quantifiers and E, but there is no way to fully express fixed-domain quantifiers with world-relative ones. Although this argues in favor of the classical approach to quantified modal logic, the translation tactic also amounts to something of a concession in favor of free logic, for the world-relative quantifiers so defined obey exactly the free logic rules.

A problem with the translation strategy used by defenders of fixed domain quantification is that rendering the English into logic is less direct, since E must be added to all translations of all sentences whose quantifier expressions have domains that are context dependent. A more serious objection to fixed-domain quantification is that it strips the quantifier of a role which Quine recommended for it, namely to record robust ontological commitment. On this view, the domain of $\exists x$ must contain only entities that are ontologically respectable, and possible objects are too abstract to qualify. Actualists of this stripe will want to develop the logic of a quantifier $\exists x$ which reflects commitment to what is actual in a given world rather than to what is merely possible.

However, recent work on actualism tends to undermine this objection. For example, Linksy and Zalta (1994) argue that the fixed-domain quantifier can be given an interpretation that is perfectly acceptable to actualists. Actualists who employ possible worlds semantics routinely quantify over possible worlds in their semantical theory of language. So it would seem that possible worlds are actual by these actualist's lights. By cleverly outfitting the domain with abstract entities no more objectionable than the ones actualists accept, Linsky and Zalta show that the Barcan Formula and classical principles can be vindicated. Note however, that actualists may respond that they need not be commited to the actuality of possible worlds so long as it is understood that quantifiers used in their theory of language lack strong ontological import. In any case, it is open to actualists (and non actualists as well) to investigate the logic of quantifiers with more robust domains, for example domains excluding possible worlds and other such abstract entities, and containing only the spatio-temporal particulars found in a given world. For quantifiers of this kind, a world-relative domains are appropriate.

Such considerations motivate interest in systems that acknowledge the context dependence of quantification by introducing world-relative domains. Here each possible world has its own domain of quantification (the set of objects that actually exist in that world), and the domains vary from one world to the next. When this decision is made, a difficulty arises for classical quantification theory. Notice that the sentence $\exists x(x=t)$ is a theorem of classical logic, and so $\Box\exists x(x=t)$ is a theorem of K by the Necessitation Rule. Let the term t stand for Saul Kripke. Then this theorem says that it is necessary that Saul Kripke exists, so that he is in the domain of every possible world. The whole motivation for the world-relative approach was to reflect the idea that objects in one world may fail to exist in another. If standard quantifier rulers are used, however, every term t must refer to something that exists in all the possible worlds. This seems incompatible with our ordinary practice of using terms to refer to

things that only exist contingently.

One response to this difficulty is simply to eliminate terms. Kripke (1963) gives an example of a system that uses the world-relative interpretation and preserves the classical rules. However, the costs are severe. First, his language is artificially impoverished, and second, the rules for the propositional modal logic must be weakened.

Presuming that we would like a language that includes terms, and that classical rules are to be added to standard systems of propositional modal logic, a new problem arises. In such a system, it is possible to prove (CBF), the converse of the Barcan Formula.

(CBF) $\Box\forall xA \rightarrow \forall x\Box A$.

This fact has serious consequences for the system's semantics. It is not difficult to show that every world-relative model of (CBF) must meet condition (ND) (for 'nested domains').

(ND) If wRv then the domain of w is a subset of the domain of v.

However (ND) conflicts with the point of introducing world-relative domains. The whole idea was that existence of objects is contingent so that there are accessible possible worlds where one of the things in our world fails to exist.

A straightforward solution to these problems is to abandon classical rules for the quantifiers and to adopt rules for free logic (FL) instead. The rules of FL are the same as the classical rules, except that inferences from $\forall xRx$ (everything is real) to Rp (Pegasus is real) are blocked. This is done by introducing a predicate 'E' (for 'actually exists') and modifying the rule of universal instantiation. From $\forall xRx$ one is allowed to obtain Rp only if one also has obtained Ep. Assuming that the universal quantifier $\forall x$ is primitive, and the existential quantifier $\exists x$ is defined by $\exists xA =df \sim\forall x\sim A$, then FL may be constructed by adding the following two principles to the rules of propositional logic

Universal Generalization. If $B\rightarrow A(y)$ is a theorem, so is $B\rightarrow\forall xA(x)$.

Universal Instantiation. $(\forall xA(x) \ \&\ En)\rightarrow A(n)$

(Here it is assumed that A(x) is any well-formed formula of predicate logic, and that A(y) and A(n) result from replacing y and n properly for each occurrence of x in A(x).) Note that the principle of universal generalization is standard, but that the instantiation axiom is restricted by mention of En in the antecedent. In FL, proofs of formulas like $\exists x\Box(x=t)$, $\forall y\Box\ \exists x(x=y)$, (CBF), and (BF) which seem incompatible with the world-relative interpretation, are blocked.

One philosophical objection to FL is that E appears to be an existence predicate, and many would argue that existence is not a legitimate property like being green or weighing more than four pounds. So philosophers who reject the idea that existence is a predicate may object to FL. However in most (but not all) quantified modal logics that include identity (=) these worries may be skirted by defining E as follows.

$Et =_{df} \exists x(x=t)$.

The most general way to formulate quantified modal logic is to create FS by adding the rules of FL to a given propositional modal logic S. In situations where classical quantification is desired, one may simply add Et as an axiom to FS, so that the classical principles become derivable rules. Adequacy results for such systems can be obtained for most choices of the modal logic S, but there are exceptions.

A final complication in the semantics for quantified modal logic is worth mentioning. It arises when non-rigid expressions such as 'the inventor of bifocals', are introduced to the language. A term is non-rigid when it picks out different objects in different possible worlds. The semantical value of such a term can be given by what Carnap (1947) called an individual concept, a function that picks out the denotation of the term for each possible world. One approach to dealing with non-rigid terms is to employ Russell's theory of descriptions. However, in a language that treats non rigid expressions as genuine terms, it turns out that neither the classical nor the free logic rules for the quantifiers are acceptable. (The problem can not be resolved by weakening the rule of substitution for identity.) A solution to this problem is to employ a more general treatment of the quantifiers, where the domain of quantification contains individual concepts rather than objects. This more general interpretation provides a better match between the treatment of terms and the treatment of quantifiers and results in systems that are adequate for classical or free logic rules (depending on whether the fixed domains or world-relative domains are chosen).

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Modal logic

From Wikipedia, the free encyclopedia

In philosophical logic, a **modal logic** is any logic for handling *modalities*: concepts like *possibility*, *impossibility*, and *necessity*. Logics for handling a number of other ideas, such as *eventually*, *formerly*, *can*, *could*, *might*, *may*, *must* are by extension also called modal logics, since it turns out that these can be treated in similar ways.

A formal modal logic represents modalities using unary modal operators. For example, "Jones's murder was a possibility"; "Jones was possibly murdered"; and "It is possible that Jones was murdered," all contain the notion of possibility; in a modal logic this is represented as an operator, *Possibly*, attaching to the sentence *Jones was murdered.*

The basic **modal operators** are usually written $\Box$ (or *L*) for *Necessarily* and $\Diamond$ (or *M*) for *Possibly*. Each can be defined from the other and negation. For example:

$$\Diamond P \leftrightarrow \neg\Box\neg P.$$

Thus it is *possible* that Jones was murdered if and only if it is *not necessary* that Jones was *not* murdered.

Contents

Alethic modalities

Necessity and possibility are sometimes called *alethic* modalities, from the Greek *aletheia*, truth. Modal logic was first developed to deal with these concepts, and only afterward was extended to others. For this reason, or perhaps for their familiarity and simplicity, necessity and possibility are often casually treated as *the* subject matter of modal logic.

A sentence is said to be

- possible if it *might* be true (regardless of whether it is or is not actually true);
- necessary if it *could not possibly* be false;
- contingent if it is *actually* true, but *not* necessarily true. It could have been otherwise, so it is possibly true, and possibly false.

Thus if something is necessarily true, then it is true; if it is true, then it is possible.

Logical necessity

There are a number of different alethic modalities: logical possibility is, perhaps, the weakest, since almost anything intelligible is logically possible: *Possibly*, pigs can fly, Elvis is still alive, and the atomic theory of matter is false.

Likewise, almost nothing is logically impossible: something logically impossible is called a contradiction or a logical falsehood. It is possible that Elvis is alive; but it is *impossible* that Elvis is alive and is not alive. Many logicians also hold that mathematical truths are logically necessary: it is impossible that $2+2 \neq 4$.

Something which is logically necessary is called a logical truth. For example, it is *necessary* that *if* Elvis is alive, *then* he is alive.

Physical possibility

Something is physically possible if it is permitted by the laws of nature. For example, it is possible for there to be an atom with an atomic number of 150, though there may not in fact be one. On the other hand, it is not possible, in this sense, for there to be an element whose nucleus contains cheese. While it is logically possible to travel faster than the speed of light, it is not, according to modern science, physically possible.

Metaphysical possibility

Philosophers ponder the properties objects have independently of those dictated by scientific laws. For example, it might be metaphysically necessary, as some have thought, that all thinking beings have bodies and can experience the passage of time, or that God exists (or does not exist). Saul Kripke has argued that every person necessarily has the parents they do have: anyone with different parents wouldn't be the same person.

Metaphysical possibility is generally thought to be stronger than bare logical possibility (fewer things are possible). Its exact relation to physical possibility is a matter of some dispute. Philosophers also disagree over whether metaphysical truths are necessary merely "by definition", or whether they reflect some underlying deep facts about the world, or something else entirely.

Confusion with epistemic modalities

Alethic modalities and epistemic modalities (see below) are often expressed in English using the same words. Thus, "It is possible that bigfoot exists" might mean either *It would be possible for such a creature as a bigfoot to exist*, or (more likely), "As far as I *know*, there may be some bigfoots."

In the former case, the speaker might know that there are not any bigfoots, but is saying that (unlike round squares), there could be some--the existence of bigfoot is not impossible. In the latter case he is saying that there may well be some *right now*.

Epistemic logic

Epistemic modalities (from the Greek *episteme*, knowledge), deal with the *certainty* of sentences. The operators are translated as "It is certainly true that..." and "It may (given the available information) be true that..." In ordinary speech both modalities are often expressed in similar words; the following contrasts may help:

A person, Jones, might reasonably say *both*: (1) "No, it is *not* possible that Bigfoot exists; I am quite certain of that;" *and*, (2) "Sure, Bigfoot possibly *could* exist." What Jones means by (1) is that given all the available information, there is no question remaining as to whether Bigfoot exists. This is an epistemic claim. By (2) he means that things might have been otherwise. He does not mean "it is *possible that* Bigfoot exists--for all I know." (So he is not contradicting (1).) Rather, he is making the *metaphysical* claim that it's *possible for* Bigfoot to exist, *even though he doesn't.*

From the other direction, Jones might say, (3) "It is *possible* that Goldbach's conjecture is true; but also *possible* that it is false," and *also* (4) "if it *is* true, then it is necessarily true, and not possibly false." Here Jones means that it is *epistemically possible* that

it is true or false, for all he knows (Goldb...n's conjecture has not been proven either true or false). But if there *is* a proof (heretofore undiscovered), then that would show that it is not *logically* possible for Goldbach's conjecture to be false—there *could be no* set of numbers that violated it. Logical possibility is a form of *alethic* possibility; (4) makes a claim about whether it is *possible for* a mathematical truth to have been false, but (3) only makes a claim about whether it is *possible that* the mathematical claim turns out false, *for all Jones knows*, and so again Jones does not contradict himself. It is worthwhile to observe that Jones is not necessarily correct: It is possible (epistemically) that Goldbach's conjecture is both true and unprovable.

Epistemic possibilities also bear on the actual world in a way that metaphysical possibilities do not. Metaphysical possibilities bear on ways the world *might have been*, but epistemic possibilities bear on the way the world *may be* (for all we know). Suppose, for example, that I want to know whether or not to take an umbrella before I leave. If you tell me "It is *possible that* it is raining outside"--in the sense of epistemic possibility--then that would weigh on whether or not I take the umbrella. But if you just tell me that "It is *possible for* it to rain outside"--in the sense of *metaphysical possibility*--then I am no better off for this bit of modal enlightenment.

Temporal logic

There are several analogous modes of speech, which though less likely to be confused with alethic modalities are still closely related. One is talk of time. It seems reasonable to say that possibly it will rain tomorrow, and possibly it won't; on the other hand, if it rained yesterday, if it really already did so, then it cannot be quite correct to say "It may not have rained yesterday." It seems the past is "fixed," or necessary, in a way the future is not. This is sometimes refered to as accidental necessity.

A standard method for formalizing talk of time is to use *two* pairs of operators, one for the past and one for the future. For the past, let "It has always been the case that . . ." be equivalent to the box, and let "It was once the case that . . ." be equivalent to the diamond. For the future, let "It will always be the case that . . ." be equivalent to the box, and let "it will eventually be the case that . . ." be equivalent to the diamond. If these two systems are used together, it will, obviously, be necessary to indicate, as by subscripts, which box is which.

Additional binary operators are also relevant to temporal logics, *q.v.* Linear Temporal Logic.

Deontic logic

Likewise talk of morality, or of obligation and norms generally, seems to have a modal structure. The difference between "You must do this" and "You may do this" looks a lot like the difference between "This is necessary" and "This is possible." Such logics are called *deontic*, from the Greek for "duty".

Other modal logics

Significantly, modal logics can be developed to accommodate most of these idioms; it is the fact of their common logical structure (the use of "intensional" or non-truth-functional sentential operators) that make them all varieties of the same thing. Epistemic logic is arguably best captured in the system "S4" ; deontic logic in the system "D", temporal logic in "t" (sic:lowercase) and alethic logic arguably with "S5".

Interpretations of modal logic

In the most common interpretation of modal logic, one considers "all logically possible worlds". If a statement is true in all possible worlds, then it is a necessary truth. If a statement happens to be true in our world, but is not true in all possible worlds, then it is a contingent truth. A statement that is true in some possible world (not necessarily our own) is called a possible truth.

Whether this "possible worlds idiom" is the best way to interpret modal logic, and how literally this idiom can be taken, is a live issue for metaphysicians. For example, the possible worlds idiom would translate the claim about Bigfoot as "There is some possible world in which Bigfoot exists". To maintain that Bigfoot's existence is possible, but not actual, one could say, "There is some possible world in which Bigfoot exists; but in the actual world, Bigfoot does not exist". But it is unclear what it is that

making modal claims commits us to. Are ... really alleging the existence of possible worlds, every bit as real as our actual world, just not actual? David Lewis made himself notorious by biting the bullet, then asserting that possible worlds are as real as our own. This position is called "modal realism". Unsurprisingly, most philosophers decline to sign on to this ontologically extravagant doctrine, preferring to seek various ways to paraphrase away the ontological commitments implied by our modal claims.

Formal rules

Many systems of modal logic, with widely varying properties, have been proposed since C. I. Lewis began working in the area in 1910. Hughes and Cresswell (1996), for example, describe 42 normal and 25 non-normal modal logics. Zeman (1973) describes some systems Hughes and Cresswell omit..

Modern treatments of modal logic begin by augmenting the propositional calculus with two unary operations, one denoting "necessity" and the other "possibility." The notation of Lewis, much employed since, denotes "necessarily *p*" by a prefixed "box" ($\Box p$) whose scope is established by parentheses. Likewise, a prefixed "diamond" ($\Diamond p$) denotes "possibly *p*." Regardless of notation, each of these operators is definable in terms of the other:

- $\Box p$ (necessarily *p*) is equivalent to $\neg\Diamond\neg p$ ("not possible that not-*p*")
- $\Diamond p$ (possibly *p*) is equivalent to $\neg\Box\neg p$ ("not necessarily not-*p*")

Hence $\Box$ and $\Diamond$ form a dual pair of operators.

In many modal logics, the necessity and possibility operators satisfy the following analogs of de Morgan's laws from Boolean algebra:

"It is **not necessary that** *X*" is logically equivalent to "It is **possible that not** *X*".

"It is **not possible that** *X*" is logically equivalent to "It is **necessary that not** *X*".

Precisely what axioms and rules must be added to the propositional calculus to create a usable system of modal logic is a matter of philosophical opinion, often driven by the theorems one wishes to prove. Many modal logics, known collectively as normal modal logics, include the following rule and axiom:

- **N**, *Necessitation Rule*: If *p* is a theorem (of any system invoking N), then $\Box p$ is likewise a theorem.
- **K**, *Distribution Axiom*: If $\Box(p \rightarrow q)$ then $\Box p \rightarrow \Box q$.

The weakest normal modal logic, named *K* in honor of Saul Kripke, is simply the propositional calculus augmented by $\Box$, the rule **N**, and the axiom **K**. *K* is weak in that it fails to determine whether a proposition can be necessary but only contingently necessary. That is, it is not a theorem of *K* that if $\Box p$ is true then $\Box\Box p$ is true, i.e., that necessary truths are "necessarily necessary." If such perplexities are deemed forced and artificial, this defect of *K* is not a great one. In any case, different answers to such questions yield different systems of modal logic.

Adding axioms to *K* gives rise to other well-known modal systems. One cannot prove in *K* that if "*p* is necessary" then *p* is true. The axiom **T** remedies this defect:

- **T**, *Reflexivity Axiom*: $\Box p \rightarrow p$ (If *p* is necessary, then *p* is the case.) **T** holds in most but not all modal logics. Zeman (1973) describes a few exceptions, such as S1^0.

Other well-known elementary axioms are:

- **4**: $\Box p \rightarrow \Box\Box p$
- **B**: $p \rightarrow \Box\Diamond p$

- **D**: $\Box p \rightarrow \Diamond p$
- **E**: $\Diamond p \rightarrow \Box\Diamond p$.

These axioms yield the systems:

- K := **K** + **N**
- T := K + **T**
- $S4$:= T + 4
- $S5$:= $S4$ + **B** or T + **E**
- D := K + **D**.

K through *S5* form a nested hierarchy of systems, making up the core of normal modal logic. *D* is primarily of interest to those exploring the deontic interpretation of modal logic.

The commonly employed system *S5* simply makes all modal truths necessary. For example, if *p* is possible, then it is "necessary" that *p* is possible. Also, if *p* is necessary, then it is necessary that *p* is necessary. This is commonly justified on the grounds that *S5* is the system obtained if every possible world is possible relative to every other world. Nevertheless, other systems of modal logic have been formulated, in part because *S5* does not describe every kind of metaphysical modality of interest. This suggests that talk of possible worlds and their semantics may not do justice to all modalities.

Development of modal logic

Although Aristotle's logic is almost entirely concerned with the theory of the categorical syllogism, there are passages in his work, such as the famous Sea-Battle Argument in *De Interpretatione* § 9, that are now seen as anticipations of modal logic and its connection with potentiality and time. Modal logic as a self-aware subject owes much to the writings of the Scholastics, in particular William of Ockham and John Duns Scotus, who reasoned informally in a modal manner, mainly to analyze statements about essence and accident.

C. I. Lewis founded modern modal logic in his 1910 Harvard thesis and in a series of scholarly articles beginning in 1912. This work culminated in his 1932 book *Symbolic Logic* (with C. H. Langford), which introduced the five systems *S1* through *S5*. The contemporary era in modal logic began in 1959, when Saul Kripke (then only a 19 year old Harvard University undergraduate) introduced the now-standard Kripke semantics for modal logics. These are commonly referred to as "possible worlds" semantics. Kripke and A. N. Prior had previously corresponded at some length.

A. N. Prior created temporal logic, closely related to modal logic, in 1957 by adding modal operators [F] and [P] meaning "henceforth" and "hitherto." Vaughan Pratt introduced dynamic logic in 1976. In 1977, Amir Pnueli proposed using temporal logic to formalise the behaviour of continually operating concurrent programs. Flavors of temporal logic include propositional dynamic logic (PDL), propositional linear temporal logic (PLTL), linear temporal logic (LTL), computational tree logic (CTL), Hennessy-Milner logic, and *T*.

The mathematical structure of modal logic, namely Boolean algebras augmented with unary operations (often called "modal algebras"), began to emerge with J. C. C. McKinsey's 1941 proof that *S2* and *S4* are decidable, and reached full flower in the work of Alfred Tarski and his student Bjarni Jonsson (Jonssón and Tarski 1951-52). This work revealed that *S4* and *S5* are models of interior algebra, a proper extension of Boolean algebra originally designed to capture the properties of the interior and closure operators of topology. Texts on modal logic typically do little more than mention its connections with Boolean algebra and topology. For a thorough survey of the history of formal modal logic and of the associated mathematics, see Goldblatt (2006). (http://www.mcs.vuw.ac.nz/~rob/papers/modalhist.pdf)

Intensionality and modal logic

Some people argue that modal logics are characterized by semantic *intensionality*: the truth value of a complex formula cannot be determined by the truth values of its subformulae, and modal operators cannot be formalized by an extensional semantics: both "George W. Bush is President of the United States" and "2 + 2 = 4" are true, yet "*Necessarily*, George W. Bush is President of the

United States" is false, while "*Necessarily*, 2 + 2 = 4" is true.

Actually, this claim is not correct, since we can give the semantics of a modal logic by structural induction, if we use stateful models, also called *coalgebraic* models. For example, we can consider the following very simple modal logic syntax:

$$F ::= \Diamond F | F \wedge F | \neg F | \text{true}$$

We can derive dual connectives using the basic ones:

$$\text{false} = \neg \text{true}$$

$$\Box F = \neg(\Diamond \neg F)$$

$$F_1 \vee F_2 = \neg(\neg F_1 \wedge \neg F_2)$$

The truth value of a formula is defined over models that are not sets, but **transition systems**.

A transition system is a pair (*S*,*T*) where *S* is a set and $T \subseteq S \times S$.

The interpretation of the logic over the state $s \in S$, given a transition system (*S*,*T*), is a relation $\models \subseteq S \times F$, where $s \models F$ is read "the state *s* satisfies the formula *F*", given by structural induction as follows:

$$s \models \neg F \iff \text{not } s \models F$$

$$s \models F_1 \wedge F_2 \iff s \models F_1 \text{ and } s \models F_2$$

$$s \models \Diamond F \iff \exists s_1.(s, s_1) \in T \text{ and } s_1 \models F$$

If we view a transition system (*S*,*T*) as a set *S* of states and a set *T* of transitions from a state to another, the modal formula $\Diamond F$, which is called the "next" modality, is read as "in my possible next states, there is one that satisfies *F*".

This logic is too simple for practical uses; more complicated logics can have more complicated models (an example being Kripke frames), however the definition of the semantics is usually given by structural induction over states.

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See also

- Accessibility relation
- De dicto and de re
- Description logic
- Dynamic logic
- Hybrid logic
- Interior algebra
- Interpretability logic
- Kripke semantics
- Possible worlds
- Problem of the futures contingents
- Provability logic

External links

- Stanford Encyclopedia of Philosophy:
  - "Modal logic (http://plato.stanford.edu/entries/logic-modal)" -- by James Garson.
  - "Provability Logic (http://plato.stanford.edu/entries/logic-provability/)" -- by Rineke Verbrugge.
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- List of Logic Systems (http://www.cc.utah.edu/~nahaj/logic/structures/systems/index.html) List of many modal logics with sources, by John Halleck.
- Advances in Modal Logic. (http://aiml.net/) Biannual international conference and book series in modal logic.

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